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TONE REPRODUCTION IN THE "HALFTONE" PHOTO-ENGRAVING PROCESS

By Herbert E. Ives

The "halftone" photo-engraving process was invented, and its technique developed, prior to the days of what is commonly called "photographic sensitometry". The necessary conditions and the appropriate operations for securing "high-light" and "shadow detail", were found by empirical studies guided by the appearance of the result, as appraised by the unaided eye. No comprehensive sensitometric study of the halftone reproduction process appears to have been made, at any rate none have been published.[1] The work about to be described is no more than a rough survey of the problem, but, due largely to the use of accurate photometric measurements, and the correlation of these measurements with other sensitometric data, rather decisive conclusions have been possible as to the essential characteristics of the process in question, and on the procedures necessary for its complete success.

This work is the outcome of a study described in a previous paper[2] (which should be consulted as an introduction to this) of the characteristics of electrically transmitted pictures. These pictures may be received in either of two forms, either as pictures of variable density or as pictures of variable sized structure. Certain of the properties of

[1] The only sensitometric study I have found is in what was apparently intended to be a series of articles describing "Experiments on the Halftone Process" by A. J. Bull, W. J. Smith, and E. L. Turner, in "The Photographic Journal," *41*, p. 8; 1917. The results obtained by these investigators are in general agreement with those here presented, although their direction of approach to the problem is quite different.

[2] "Some Photographic Problems Encountered in the Transmission of Pictures by Electricity," Ives, J.O.S.A. & R.S.I., March, 1926.

variable sized structure pictures, which were brought out by that study, are shared by the dot structure of the halftone picture. Consideration of these properties raised certain questions as to the fidelity of tone reproduction in the halftone process as ordinarily carried through, and the investigation of these questions led to the accumulation of data here used.

The results of this study may be summarized in the statement that the practically universal procedure of making a dot negative directly from the object or copy by exposure through a cross-line screen does not yield correct tone values. A procedure is indicated, however, involving additional photographic operations, by which a dot picture of correct tone characteristics may be produced.

General Statement of Conditions for Correct Tone Reproduction: As was pointed out in the earlier paper referred to, a negative in which the lights and shades are represented by a structure of constant density but variable size (e.g. lines or dots) differs from a variable density negative in the nature of the print which can be made from it. The print from a structured negative is *complementary* in its transmission to the negative; the opaque structure of the negative is reproduced as a transparent structure in the print, and vice versa. On the other hand, a print from a variable density negative is, generally speaking, of a *reciprocal* character with respect to transmission.

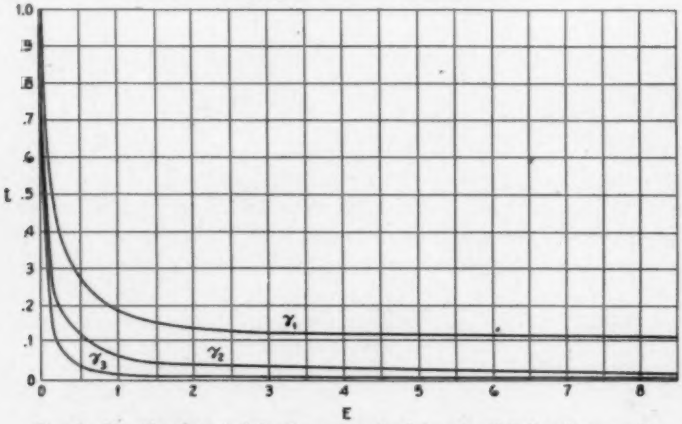


FIG. 1. *Negative characteristics for various developments, plotted on linear scale.*

The conditions for correct tone reproduction with variable density negatives have been very completely developed and need not be given

in detail here.[3] It will suffice for our present purpose to show a group of characteristic curves, that is relationships between exposure and transmission of the developed negative. These, plotted on a linear scale, are shown in Fig. 1. When printing media of approximately the same properties are used, the very pronounced curvatures, which the characteristics of Fig. 1 show, are opposed or matched, and the result of a second or printing operation will, under proper conditions, yield a characteristic of a linear type. This is shown in Fig. 2, where

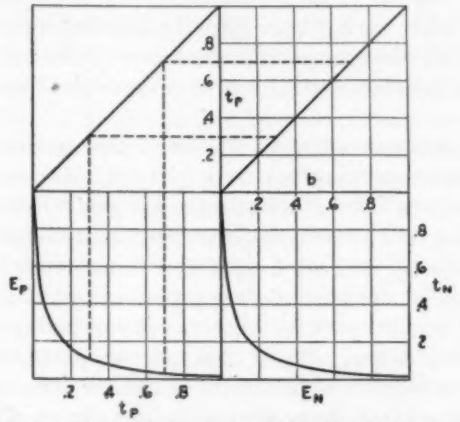


FIG. 2. *Conditions for correct tone rendering, plotted on linear scale.*

the characteristics of successive operations are shown plotted in the several quadrants into which the figure is divided. This method of plotting is essentially that used by L. A. Jones,[3] who has employed it with logarithmic (Hurter and Driffield) plots, in which form it will occur later in this paper. In the linear form here used, the fourth quadrant shows the negative characteristic, with the symbol t_N for the transmission of the negative and E_N for the exposure, which is proportional to b, the brightness of the original object. The third quadrant gives the characteristic of the positive or printing medium, the symbols t_p and E_p, which refer to the transmission and exposure (proportional to t_N) indicating by their orientation the direction in which the corresponding scales are to be read. The second quadrant contains merely a line at 45°, used as an aid for projecting the values of t_p back to the first quadrant, where in combination with the value of b they fix the

[3] See L. A. Jones, "Photographic Reproduction of Tone," J.O.S.A. & R.S.I., *5*, p. 3; p. 232, 1921.

curve showing the relationship between original object brightness and positive transmission. Correct tone reproduction is attained when the relationship between original object brightness and the transmission or reflecting power of the print, is given, on the linear scale used in this figure, by a straight line in the first quadrant passing through the intersection of the axes of the diagram. Various inclinations of this line (up to 45°) correspond to different relative illuminations of the original object or call for different illuminations of the print to make it match the object. In the simplest case, where the original object is a black and white picture on a white background, perfect tone reproduction is attained when the reflecting powers of the final photographic print plot against the reflecting powers of the original copy, as a straight line at 45°.

It is customary in scientific studies of photographic processes to plot the characteristics which have just been discussed in the logarithmic form originated by Hurter and Driffield. The coordinates are the logarithms of exposure, and the logarithms of the reciprocals of the transmissions; the latter quantities being termed the densities. The condition for final correct tone reproduction remains unchanged, that is, that the characteristics of negative and positive should match or oppose each other. Since, however, the photographic sensitive medium characteristics when plotted in this logarithmic form consist, for a large part of their range, of straight lines, the condition for correct tone reproduction becomes simply that the product of the slopes of these straight lines (development factors) should be unity. The plotted relationship between object brightness and reproduction brightness (reflecting power or transmission) should therefore be a straight line at an angle of 45°. The various straight lines at different angles which represent correct tone reproduction on the linear plots, are, on the logarithmic plots, a family of straight lines all at 45°, but displaced relatively to each other.

The Hurter and Driffield plot is an extremely useful and instructive manner of representing photographic phenomena, and works out, because of the long straight line portions in most photographic materials, very conveniently in studying variable density pictures. In the case of structured pictures, however, we are never concerned with the extreme range of transmission values which may be found in ordinary photographic negatives. Our tones must all lie between values of transmission representing the practical white and black of the printed page. Moreover, as will be seen, the straight line, which is a

prominent feature of the Hurter and Driffield plot in variable density pictures, does not occur in the logarithm plot of variable sized structure pictures. An adequate study of halftone characteristics might well be made by the use of linear plots exclusively. However, in order to correlate the requirements for halftone reproduction with the vast body of photographic information which has been recorded in terms of Hurter and Driffield coordinates, use will be made in the present discussion of both types of plotting.

The Ideal Characteristic for a Halftone Negative: When a halftone printing plate is brushed over with finely powdered chalk, leaving the printing surfaces bare, the picture appears as a positive, corresponding to the impression made upon paper from the inked plate. The raised unetched surfaces on which the ink is applied are the places which have been acted on by light and protected from the etching operation. Consequently the photographic image which must be used to print on the metal must be transparent in these portions, that is, it must be a negative, in the sense that the blacks of the final picture are its clear spaces, and the whites its opaque spaces. It is, of course, a structured negative; the various lights and shades being represented by structures of various sizes but equal density. It must furthermore, as already pointed out, be complementary in its transmissions to the reflecting power desired in the final print. In other words, it must be complementary to the original object.

What is the form of the photographic characteristic curve demanded, plotted on linear coordinates? The characteristic is quite simple; it

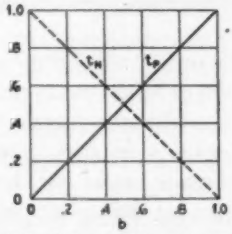


FIG. 3. *Ideal characteristic of structured negative (dashed line); linear scale.*

consists of a straight line. This is illustrated in Fig. 3, where against the reflecting powers of the original object are plotted, in the full line, the reflecting powers of the reproduction which is ultimately aimed for, the brightness and reflecting power data running from zero to unity.

In the dashed line is shown the characteristic required of the negative, which is simply the complementary of the characteristic first given.

Comparison of this straight line with the characteristic shown in Fig. 1, indicates at once that a negative of very different character from ordinary structureless negatives is required. The nature of this difference is again shown, in terms of the Hurter and Driffield plot in Fig. 4, where densities, that is, logarithms of the reciprocals of the

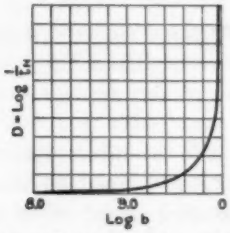


FIG. 4. *Ideal characteristic of structured negative, logarithmic scale.*

transmissions, are plotted against logarithms of object brightness. As has already been anticipated, the ideal structured negative characteristic is altogether different from the straight line which occurs so commonly in variable density photographic reproduction. The negative which is called for is, as the figure shows, one possessing an extremely curved characteristic quite unlike that of any ordinary photographic material. It is our immediate problem to determine whether the halftone dot negative as ordinarily produced conforms to this characteristic, and if not, to find under what conditions a negative of this characteristic might be produced.

Experimental Study: In order to investigate the characteristic of the halftone negative as produced in the usual commercial method, photographs were made of a uniformly illuminated neutral tint wedge whose transmission values ran from a maximum to approximately 1% of this. Negatives were made under our direction by a commercial photo-engraving establishment using, in the majority of cases, the collodion wet plate process. Our chief interest lay in the characteristic as produced by a single exposure with no resort to variation of stop size, "flashing" or other operations. This requirement pre-supposes that some set of conditions, that is, choice of lens stop or aperture, distance between the crossline screen and plate, exposure, and development, does exist for which clearly defined dots may be produced over

the whole range of image brightnesses which are to appear in the final reproduction. Such did prove to be the case and the chief results given below apply to conditions, which are stated, under which a record of all tone values in the original over a range of approximately 100 to one are represented by definite dots.

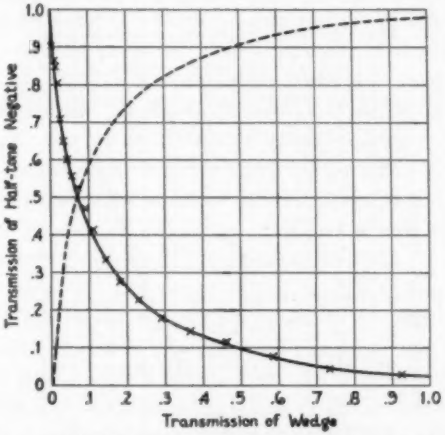


FIG. 5. *Actual halftone negative characteristic, on linear plot. Dashed line shows characteristic of print from this negative.*

The experimental conditions for producing the wet plate dot negatives conforming to the requirements as above set forth consist in the use of an F–16 stop, with a separation between screen (65 lines to the linear inch) and plate of 3/8″. Exposures were chosen such that in the most exposed portions the dot almost but not quite filled in, corresponding to a transmission of about 1 per cent. The highest transmissions with these conditions were 90 to 95 per cent. A smaller range would be preferable for practical halftone work, but this can readily be attained under conditions which do not affect the principal consequences of this paper.

The negatives as developed by the regular commercial procedure were measured on a Martens polarization photometer, and the results of representative exposures are shown in Figs. 5–6, plotted on both the linear and logarithmic coordinates. It will be seen by reference to the linear plot, that the characteristic is not a straight line, but a very pronounced curve, widely differing from the kind of negative necessary for correct tone reproduction in a structured picture as shown in Fig. 3.

The complementary curve shown by the dashed line, indicates the very distorted tone reproduction which a positive print from this negative would give. We see by reference to the logarithmic plot that the characteristic approximates much more nearly to an ordinary variable density negative characteristic on this plot, that is to a straight line, than to the pronounced curve demanded for our purpose.

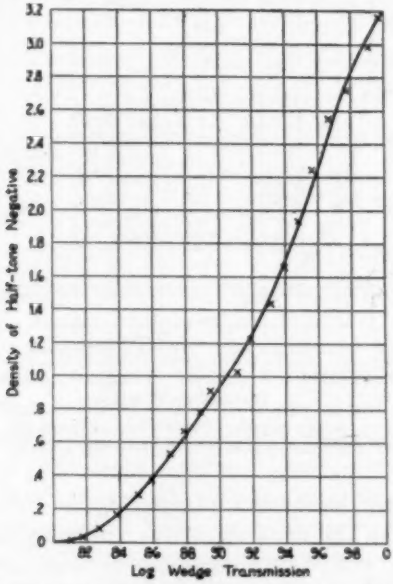


FIG. 6. *Actual halftone negative characteristic, logarithmic plot.*

However it is definitely not a straight line on the Hurter and Driffield plot. The shape actually is such as to be very well represented by the equation:

$$\sqrt{D}=a+b \log E$$

This characteristic, which was obtained uniformly with wet plates under the conditions described, is also found to hold with "process" dry plates. In order with the latter to secure dots over the same range of exposure, it is necessary to work with somewhat smaller lens diaphragms, thus offsetting the poorer approximation to an "all or more" action of the dry plate emulsion.

In view of the wide departure of this characteristic from the ideal, the question at once arises, how is accuracy of tone reproduction at-

tained in the many thousands of halftone reproductions used in the modern illustrated book or periodical? The answer to the question is somewhat complicated. In the first place, it may be pointed out that a great deal of halftone reproduction is not by any means faithful. For instance, in many newspaper illustrations the light grays and whites are rendered as of the same uniform value, the characteristic of a representative newspaper halftone being in fact of the general nature of the dashed curve of Fig. 5. This is actually a practical advantage in printing with poor papers and inks from stereotype copies of the original etched plates. In the second place, halftone negatives are not ordinarily made by straightforward uninterrupted exposure to the original such as has been here adopted for purposes of study. Resort is made to two expedients in the exposure, namely, the use of multiple stops and the process of "flashing", both of which may be made to warp the characteristic toward the shape desired.

Some discussion of the processes of multiple stop exposure and flashing, from the viewpoint of this paper, is in order: First of all, it is of importance to know the effect of changing the size of the lens diaphragm or stop. As an extreme case, the stop may be considered so large that with a given finite separation of screen and plate, the illumination of the sensitive plate is practically uniform. Under these conditions we obtain the normal characteristic of the wet plate process. This is shown in Fig. 7, curve *A*. It consists of a very steep, approximately straight line. Its approximation to an "all or none" response is the property which fits it peculiarly for the production of structured images from the shaded dot image produced by the cross-line screen, and it is this peculiarity which dry plates have difficulty in copying. As the diaphragm size is reduced a point is reached where dots of large size are produced; but the image forming the dot is too uniform in intensity over its area to permit the formation of small dots by shorter exposures. The picture then consists (approximately) merely of portions in which large dots are produced, the rest of the picture being unformed. The characteristic curve produced by such an exposure is approximately that shown in Curve *B*, of Fig. 7. At the other extreme of dot formation, is the characteristic obtained when a very small stop is used. This gives, behind the screen, a series of small images which very quickly attain a saturation value of exposure but cannot increase much in size. The characteristic of an exposure under these conditions is represented approximately by Curve *C*, of Fig. 7. Still another dot characteristic is that obtained from the process of

flashing, that is, exposing, with a small stop, to a uniform field such as a white card. A characteristic of such an exposure is given in Curve *D*, of Fig. 7. It is, of course, a straight line parallel to the axis of abscissae.

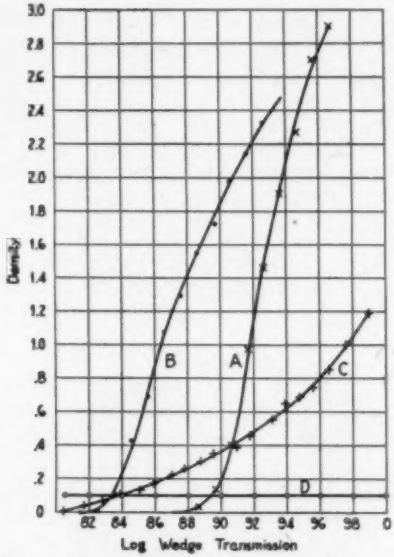


FIG. 7. *Characteristic of collodion dry plate. A*, without screen; *B*, through screen, with large lens diaphragm; *C*, through screen, with small lens diaphragm; *D*, characteristic obtained by "flashing" with a small diaphragm.

A point to bear in mind with reference to structured pictures is that if a large structure has been impressed, the subsequent exposure to a small structure does not produce any effect. For instance, a large dot when formed is not altered by the production of a small dot within its limits. As a result of this, it is possible to obtain on the one plate portions of the characteristics due to different conditions of exposure. As an illustration, in Fig. 8, if the first exposure is with a large stop as shown at *a*, a second exposure with a small stop, as shown at *b*, may be superposed and will have no effect except for the lower brightnesses, and similarly the flash exposure *c*, may again be superposed and only show its effect in the production of dots corresponding to the lowest brightnesses of the objects photographed. The final result will then be the envelope of the three separate curves, as shown by the heavy line. Comparison of Fig. 8 with Fig. 4 shows how

it is possible by the manipulation of stops and flashing process to approximate the ideal characteristic postulated early in the paper. This discussion could, of course, have been carried through in terms of linear plots, leading to similar conclusions as to the technique of the common processes of exposure to several stops and flashing. It is sufficiently evident that correct tone rendering in dot negatives by the ordinary expedients must be a complicated procedure, and one apt to depend for its success upon the skill and judgment of the operator rather than upon any routine capable of easy scientific control. The final result can hardly be expected (from the standpoint of accurate sensitometry) to be more than an approximation.

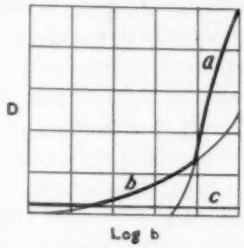


FIG. 8. *Result of successive exposures with two sizes of diaphragms, and a flash exposure.*

A Suggested Procedure for Correct Reproduction in the Halftone Process: The preceding work having indicated that the straightforward process of halftone negative making cannot yield correct negatives, attention was turned to the possibility of some variation in procedure which would produce negatives of the desired type. It was thought, in the preliminary stages of the work, that the characteristic of the halftone negative would be substantially that of an ordinary variable density negative, that is, a straight line on the Hurter and Driffield diagram. Had this been the case, a procedure would have been indicated immediately, suggested by the two matched processes of ordinary photography. This procedure would have been to introduce two extra processes; first, the production of an ordinary variable density negative, and second, to introduce the dot by exposing a wet plate, behind the screen, to the negative just described. This would result in a positive dot image. From this in turn a negative could be made by contact printing (a process which can be repeated theoretically indefinitely with structured images, without loss in quality) and this negative used for printing on metal.

The perfection of tone reproduction which would be possible by this procedure—if the initial condition were met—warrants the search for some means of straightening the characteristic, as plotted on the Hurter and Driffield diagram. (Obviously this promises to be easier than the drastic change of shape which would be required to make a straight line of the characteristic on the linear plot (Fig. 6)). One means which suggests itself is the use of lens stops which will restrain the photographic action in the peripheral portions of the dots. Star shaped apertures, or aperture screens more transparent at the center than at the edge are indicated. Trials of star shaped apertures however have not been promising, as the straightening of the characteristic, even with quite extreme star contours is quite inadequate. Other means may be found, but nothing has so far given really promising results.

The fact that the wet plate dot negative characteristic is not a straight line on the Hurter and Driffield plot and cannot be made approximately straight by an easy means rules out the suggested procedure in so far as using what would ordinarily be termed "correct negatives" is concerned. [It may be remarked that the tone reproduction which this procedure would yield using a "correct" variable density negative (in the Hurter and Driffield sense) would, in spite of its defects, be vastly better than that yielded by introducing the dot structure at the first exposure, as is now done.] However, the procedure which would be gone through were the dot negative characteristic straight, becomes a perfectly practical one provided variable density negatives can be made whose characteristics are curved on the Hurter and Driffield plot to such an extent as to match the rather slight curvature of the wet plate characteristic. This proves to be quite possible. There are quite a number of photographic emulsions which possess rather long curved characteristics and there are certain developers, for instance, metol, which produce in the relatively under exposed portions of almost any negative emulsion, a curvature very closely of the type required.

In order to illustrate this possibility, Fig. 9 has been drawn, using the four quadrant diagram of Jones. This gives, in the fourth quadrant, the characteristic of a negative of the variable density type so handled as to give a curved in place of a straight line characteristic, in the third quadrant a wet plate negative (which is now a positive), and, in the first quadrant the result of matching these two, compared with the original object. This latter plot represents the characteristic of the print which would be obtained if a contact negative from the wet

plate dot picture were used to print on a metal plate from which a typographic print were drawn. (It may be pointed out that a very simple way to determine when the dry and wet plate negatives are matched is to plot the characteristics on translucent paper and fold the diagram over on a vertical line). It will be noted that using, as

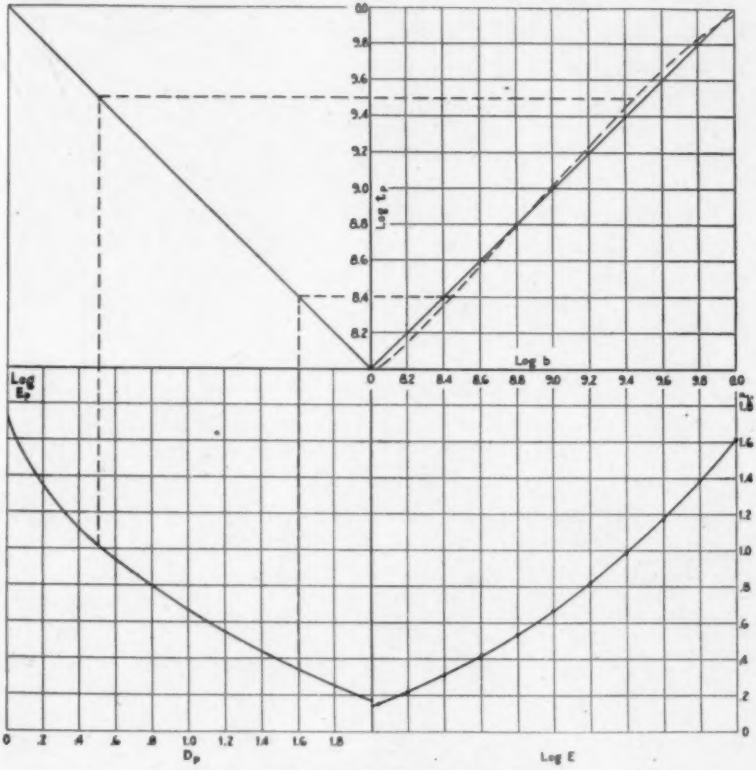


FIG. 9. *Illustrating method for correct tone reproduction in structured pictures, by making a variable density negative as the first step.*

was here done, "portrait" film developed with metol for making the first negative, the wet plate picture as obtained is a very faithful reproduction of the original, the dashed characteristic curve in the first quadrant being very accurately a straight line at 45°.

A word may be said on some practical questions which come up in producing a correct dot negative by this procedure. It is evident

that the processes of dry plate and wet plate making involved, should be very accurately standardized as to exposure and development. The density range in the dry plate negative must be such that with the proper exposure to it, the density range in the wet plate positive is exactly that required in the final result. For extreme accuracy in tone reproduction, the curvatures of the two characteristics must, of course, match accurately and further investigation than has been here reported, would undoubtedly be called for to determine the best emulsion and development to use. It may be remarked, however, that the fidelity of tone reproduction shown in Fig. 9 is of a very high order, as photographic tone reproduction goes, and is infinitely better than can be obtained by the ordinary procedure.

A practical point deserving discussion is the requirement in relief typographic printing plates of a definite dot structure at the high-light end of the tone range, in order to prevent the paper from pushing down into the hollows corresponding to considerable white areas. This requirement can be very easily met in the procedure here suggested, by "flashing" the wet plate exposure, with negligible distortion of the tone scale except at the extreme high-light end. Flashing the variable density negative, which is made as the first step, will insure a structure in the deep shadow, if this is considered necessary. These points are mentioned to show that the process here proposed has great flexibility; practically any desired deviations from straightforward faithful tone rendering which may be required for practical reasons, are readily possible. These are however to be clearly differentiated from the expedients which are now resorted to in halftone negative making for a quite different reason, namely to secure an approximation to faithful tone rendering which otherwise would be impossible.

A final matter which needs mention, is the fact that this study has concerned itself solely with the production of the structured negative of correct characteristics. No attention has been paid to the etching and printing processes and the influence they may have on the character of the final result. It is assumed that these processes are, or can be made, essentially faithful in their rendering of the structure recorded in the dot negative, so that the key problem is the attainment of correct tone rendering in that negative.

Applications: The procedure for obtaining correct halftone negatives which is here indicated involves two extra operations, namely, the production of an ordinary variable density negative as the first step,

and the contact printing of a negative from the wet plate dot positive image (this second step might be replaced by a reversing process, or a metal printing process of complementary character might be found and used). These additional steps are to be matched off against the manipulation of stops and multiple exposures now employed. It is to be expected, however, that with an art so far developed and widely practiced as that of halftone negative making, the use of a special procedure of the kind indicated may hardly be expected to be generally adopted unless the demand for accurate tone reproduction is very great. In ordinary black and white reproduction, it may be admitted that the product of the skillful workman is usually quite adequate and the advantage of shorter time for the completed printing plate probably lies with the procedure in common use.

The situation is, however, quite different in one form of halftone work, namely, three color reproduction. Here it is imperative that the three color plates all be correct in their tone relations. While any one of the three prints alone, as a monochrome picture, might appear perfectly satisfying, any deviation from an accurate linear tone relationship means that the mixing proportions of the three colors are altered and the color in the final picture distorted. It would appear, therefore, that the procedure which has been indicated in this paper is exactly what is called for in order to systematize three color halftone reproduction. It is of interest in this connection to recall that there has been a long standing disagreement between "practical" three color printers and the scientific exponents of color photography as to the colors of the inks to be used, and as to the adequacy of three color analysis to produce faithful results. The latter question is answered unequivocally by the success of the additive methods of three color photography, and of the subtractive processes as carried out in superposed dyed transparencies, indicating clearly that the difficulties which have been encountered in three-color halftone process are to be sought for in the methods of producing the printing plate and of printing the inked impression from it.[4] The study of halftone reproduction which is here presented suggests a procedure for the production

[4] Before the introduction of panchromatic "process" dry plates the structured negatives for three-color work had to be made by the wet plate process. This involved making first a set of color selection negatives on panchromatic dry plates. But it appears that the common procedure was to make the dot negatives from photographic *positives*, printed from the selection negatives, whereas, according to the results here presented, making the screened exposure on the negatives would have been preferable. (The number of photographic copyings would have been the same.)

of three-color halftone printing plates which should lend itself to the straightforward production of plates which, without elaborate reetching, should be available for use with the printing inks indicated by three color theory.

BELL TELEPHONE LABORATORIES, INCORPORATED,
NEW YORK, N. Y.,
MARCH 22, 1926.

Radiation from the Carbon Arc. The bureau is investigating the radiation from the carbon arc, a matter of importance in the treatment of diseases by exposure to light, especially sunlight. However, sunlight can not always be obtained, hence the demand for an artificial source approaching sunlight in its characteristics.

The investigation is being made in duplicate: (1) By mapping the ultraviolet spectrum by means of a quartz spectroradiometer, and (2) by measuring the spectral components of the total radiation emitted by the arc, by using a thermopile and screens which completely absorb certain spectral regions and freely transmit others.

Thus far studies have been made of the standard carbons on the market, viz, "white flame," "red flame," "yellow flame," "blue flame," and "neutral core" carbon electrodes; also several special carbons with cores of nickel, tungsten, etc. The effect of varying the current has been studied, using 15, 30, 60, 90, and 122 amperes.

The high-intensity arc (120 amperes) has been found to be closest to the sun in spectral composition. It emits considerable radiation of wave lengths longer than 4μ, which are not in the solar beam, but this can be eliminated easily by using a window of fused quartz, which absorbs the long infrared rays. [Technical News Bulletin of the Bureau of Standards, No. 111, 1926.]

Control of Optical Surfaces.—In this, the third installment of the series, the tangential profile is discussed, the normal profile having received attention in a former installment. The tangential profile is the derivative of the normal profile, and it expresses the position of the rays in such a way that the transverse errors are readily obtainable. It is with these errors that the properties of the surfaces and those of the image may be simply expressed, and the relationships between them brought out. Finally, it is shown that the tangential profile may be obtained by means of the well-known Hartmann method, and by the ingenuous new method of Cotton. Although these two methods are related in theory the experimental procedures are very different. The paper is full of detailed discussion of the subject. [G. Yvon, Revue d'Optique, *4*, pp. 181–210; 1925.]

G. W. MOFFITT

THE TRANSPARENCY OF OCEAN WATER AND THE VISIBILITY CURVE OF THE EYE

By E. O. Hulburt

ABSTRACT

Measurements of the absorption of ocean water for monochromatic light throughout the visible spectrum are recorded and show that the sea water is about as transparent as pure water. When daylight is passed through 40 meters of water the resulting spectral intensity curve is very similar to the visibility curve of the eye both as regards shape and the position of the maximum. Early life on the earth developed in a watery environment with a vaporous atmosphere. This permits the suggestion that the visibility curve of the eye owes its general characteristics to the spectral intensity curve of Palaeozoic daylight.

The great transparency of the water of the ocean, at least under normal conditions in calm weather, has long been recognized,[1] but no exact measurements of the light absorption of sea water appear to have been made. Recently in this laboratory the light absorption coefficient α of ocean water has been determined for monochromatic light throughout the visible spectrum. For a specified wave length, α is defined by the relation $I = I_0 \times 10^{-\alpha d}$ where I_0 is the original intensity of the light and I is the intensity after passing through a thickness d cms of the water.

Three samples of water from the surface of the open sea far from land were obtained, one from the Pacific Ocean, one from the Gulf Stream and one from the Caribbean Sea. These were kept in the dark in large glass bottles sealed with wax and were examined about three months after being taken from the ocean. A photometer of the Nutting type and a glass prism spectroscope were used for the light absorption measurements. A mercury lamp served as a source of light for wave lengths 436 and 546mμ and a 1000 watt tungsten lamp for the region from 500 to 700mμ. The water was in pyrex glass tubes, the longest being four meters in length. No difference could be detected between the three samples of sea water and only a very small difference between the sea water and doubly distilled fresh water, the sea water being very slightly more opaque than the pure water for wave lengths below 500mμ. The values of α, plotted as ordinates against the wave lengths λ as abscissas, are shown by the branch curve 1′, Fig. 1, which joins the curve 1 for pure water at λ550mμ. The values of α for pure water, shown

[1] Tyndall, "Fragments of Science," 1870;
Bancroft, Journal of the Franklin Institute, 1919.

by small circles along curves 1, have been taken from the measurements of others.[2] It must be mentioned that the measurements of the light absorption coefficient in the blue spectral region of great transparency are open to considerable error, 50 per cent or more. To improve the accuracy would have necessitated much longer tubes of water. The fact that the sea water is just about as transparent as pure water is to be expected when one remembers that the inorganic salts[3] in the sea all give colorless solutions.

Since the human eye has developed under daylight radiations one might naturally suppose that its spectral sensitivity curve, i.e. the visibility curve, would be similar to the spectral intensity curve of daylight. This, however, is not the case for the two curves are found to be rather dissimilar. The visibility curve of the light adapted eye of the average normal person is shown in curve 2, Fig. 1, in which the

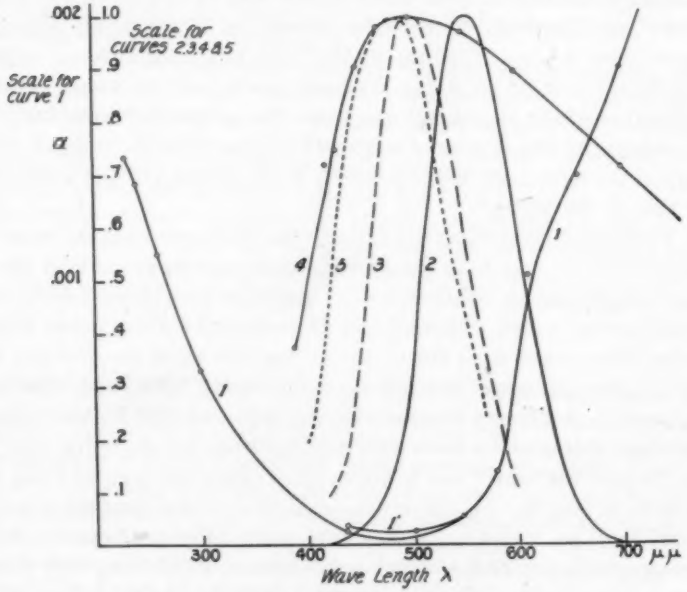


FIG. 1. *Transmission of water and visibility of the eye.*

[2] Kreusler, Ann. d. Physik, *6*, p. 412; 1901;
Aufsess, Ann. d. Physik, *13*, 1904;
Hulburt, Journ. Phys. Chem., *21*, p. 150; 1917;
Martin, Journ. Phys. Chem., *26*, p. 471; 1922.

[3] Clark, "The Data of Geochemistry," Bull. 770, U. S. Geological Survey, 1924.

relative intensity of the sensation caused by monochromatic light is plotted, with maximum ordinate arbitrarily unity, against the wave length of the light as abscissa. This curve is from the data of a number of observers;[4] the curves of the various observers were concordant, the differences for the most part being barely perceptible on the scale of curve 2. The visibility curve of the dark adapted eye,[5] curve 3, Fig. 1, is very closely curve 2 shifted to shorter wave lengths by about 50mμ. The visibility curves of the eyes of beasts and fishes have not been determined directly in many instances, but may be derived in a manner, apparently justifiable, from measurements of pupillar reaction or the light absorption of the visual purple.[6] In such ways the eyes of birds, mammals and amphibians are found to be about the same as the curve 3 for the dark adapted human eye, the maximum of which lies about 500mμ. In the case of the fishes the visibility curve appears to be similar to curve 2 which is for the light adapted human eye.

The spectral intensity curve of the light of the sun at the surface of the earth falling normally to the earth through a cloudless, but not necessarily hazeless, atmosphere[7] is given by curve 4, Fig. 1. The maximum ordinate of curve 4 is arbitrarily made unity. It is seen that although the visibility curves of the eye and the sunlight curve rise to maxima at approximately the same wave length, the eye is remarkably insensitive to red light, wave lengths from 600 to 700mμ, where the sun's radiation is still very intense.

If the light from the sun passed through 40 meters of water (it makes little difference whether the water be salt or fresh) the spectral energy curve is found to be given by curve 5, Fig. 1. Although the general trend of curve 5 is unmistakable its exact shape is none too certain because it is derived from the water absorption curve 1 which is not very precise for wave lengths from 400 to 500mμ; in particular, there are no observations between λ434 and 300mμ to guide the course of curve 1 in this region. With this uncertainty in mind it is seen that curve 5 is very similar to the visibility curve of the eye both as regards the

[4] Ives, Journ. Frank. Inst., *188*, p. 218; 1919;
Hyde, Forsythe and Cady, Astrophysical Journ., *48*, p. 65; 1918;
Tyndall and Gibson, J.O.S.A., & R.S.I., *9*, p. 403; 19 25;
Kingsbury, Phys. Rev., *7*, p. 161; 1916.

[5] Hecht and Williams, Journ. of Gen. Physiology of the Rockefeller Institute for Medical Research, *5*, p. 1; 1922–23.

[6] Nagel, "Handbuch der Physiologie," *4*, p. 99; 1904.

[7] Abbott, Fowle and Aldrich, Annals of the Astroph. Observatory of the Smithsonian Institution, *3*, p. 197; 1913.

shape and the position of the maximum. In the Proterozoic and Palaeozoic ages when life was beginning on the earth, the earth was warmer than it is now and was surrounded with water vapor, perhaps more dense than the clouds of the present time. Therefore the sunlight which fell through to the creatures living in those times was robbed of its red wave lengths either by the water in which they lived or by the mists and vapors of the atmosphere above, or both. We may take it that the spectral energy curve of Palaeozoic daylight was roughly that of curve 5. If we assume that the eyes which were formed during those periods developed visibility curves characteristic of the light, we may suggest that the visibility curve of the eye would be much the same as it is today. A discussion of the more refined questions as to the differences in the positions of the maxima of the curves 2, 3 and 5 is beyond the province of the present paper.

NAVAL RESEARCH LABORATORY,
WASHINGTON, D. C.,
APRIL 20, 1926.

An Apparatus for Measuring the Thermal Expansion of Glass and Wire. The sample on which the measurement is to be made is drawn into a fine thread or wire, each end terminating in a ball of glass. The specimen is supported vertically at its upper end by an inverted tube of quartz shaped like a test tube, the end of which is pierced; the lower end supports a similar smaller tube of quartz in a similar manner. The open ends of the two tubes are provided with an attachment with the aid of which it is possible, by means of a measuring microscope, to measure their relative displacements when the sample is heated. The outer larger tube supports an adjustable horizontal mirror with its silvered surface upward. The inner smaller tube supports a stirrup which can be loaded so as to hold the specimen taut and a clamping device for holding a piece of razor blade with its sharp edge horizontal and immediately above the silvered surface of the horizontal mirror. In this manner it is possible to observe with a measuring microscope both the sharp edge of the blade and its inage in the mirror. Thus the shift can be measured when the specimen is heated or cooled. The method of determining the expansion coefficient is described and the sources of possible error discussed. The authors state that the accuracy of the result is approximately 1 per cent and mention that in carefully controlled experiments the measured expansion on heating and contraction on cooling agree within 2 per cent. There appear to be no systematic errors and in several years of use the apparatus has given satisfactory results. [P. Selényi and E. Patai, Ujpest (Hungary) ZS. f. Instrk., *46*, pp. 246–249; 1926.]

PAUL E. KLOPSTEG.

THE OPTICAL CONSTANTS OF MOLYBDENITE IN THE ULTRAVIOLET

By Alfred W. Meyer

When plane polarized light is obliquely incident upon the polished surface of an absorbing medium, the reflected ray, in general, is elliptically polarized. From the elements of this ellipticity, the optical constants, index of refraction and coefficient of absorption, should be uniquely determined. Polished surfaces of bulk metals however do not give consistent results. Drude and others have used the surfaces of natural crystals and have obtained results which are not only consistent but are fundamentally characteristic of the materials examined, since account is taken of the crystal properties.

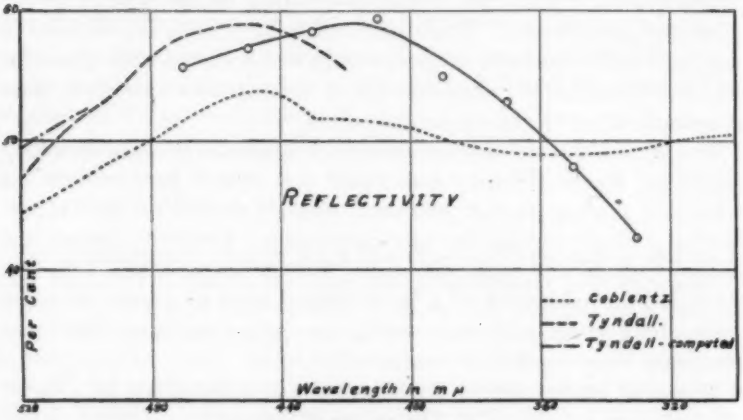
Drude[1] and Mueller[2] have obtained the constants of lead and antimony sulphides, Weld,[3] Skinner,[4] Van Dyke[5] and Miller[6] have studied the constants of selenium and tellurium. Graber[7] studied the optical properties of single crystal zinc and magnesium. Tyndall[8] obtained the optical constants of molybdenite and stibnite in the visible and also measured the reflectivity by a direct method using a Lummer-Brodhun spectrophotometer and these results are quite consistent with those computed from the optical constants.

Coblentz[9] also measured the reflectivity of molybdenite by a direct method using a thermopile. The results of Tyndall and Coblentz are in good agreement.

The work reported is a determination of the constants of molybdenite for the wave lengths included between 500 and 330 mμ. The materials used were crystals obtained from the state of Washington. The observations were made on thin cleavage surfaces perpendicular to the optic-axis. The apparatus used was the crystelliptometer constructed by Weld [10] after the method of Voigt.[11] The apparatus and method were in

[1] P. Drude, Ann. D. Phys. u. Chem., *34*, p. 849; 1888.
[2] H. Mueller, Dissert., Gottingen, 1903.
[3] L. D. Weld, J.O.S.A. & R.S.I., *6*, p. 67; 1922.
[4] C. H. Skinner, Phys. Rev., *9*, p. 148; 1917.
[5] G. D. Van Dyke, J.O.S.A. & R.S.I., *6*, p. 917; 1922.
[7] M. E. Graber, Phys. Rev., *61*, p. 380; 1925.
[8] E. P. T. Tyndall, Phys. Rev., *21*, p. 162; 1925.
[9] W. W. Coblentz, B. S. Sci. Pap., No. 493; 1924.
[10] L. D. Weld, loc. cit.
[11] W. Voight, Physik. ZS. *2*, p. 203; 1001.

general the same as those used by Weld, Van Dyke, Miller and Graber. In the present investigation a special carbon source of high ultraviolet content was used and a Hilger, Quartz prism, constant deviation, wave length spectroscope was used as a monochromator. The experimental procedure and calculations in obtaining the elements of ellipticity were similar to those of the above mentioned investigations. The equations of the optical constants as functions of the elements of polarization are given by Drude[12] and these were applied in a manner similar to that used by Tyndall.[13]



Reflectivity of molybdenite.

RESULTS

The elliptic elements are functions of the angle of incidence; and observations could be taken at various angles for the same wave length. The final results should be constant for the different angles. Observations were taken at settings of 65°, 70°, and 76° and averaged. Individual observations varied about 5 per cent from the average.

The accompanying table gives the average results for n, k, and R, the index of refraction, absorption index, and reflectivity, respectively, for each wave length setting. The curves are graphic representations of these results.

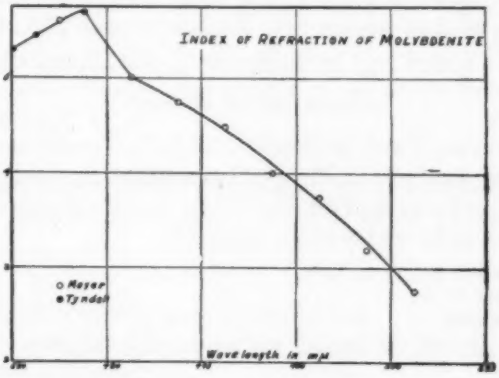
The index of refraction falls steadily throughout the range investigated. The absorption index increases steadily and the reflectivity rises to a peak and then decreases although it remains quite high over the

[12] P. Drude, Ann. d. Phys. u. Chem., *32*, p. 584; 1887.
[13] E. P. T. Tyndall, loc. cit.

TABLE 1. *Optical constants of molybdenite*

Wave length	k	n	R
500	.341	5.668	52.97
470	.468	4.999	55.09
450	.581	4.675	56.75
430	.657	4.500	58.03
410	.812	4.002	59.71
390	.888	3.853	55.04
370	.963	3.209	52.93
350	.982	2.640	47.13
330	1.001	2.217	42.10

entire range. This high reflectivity is due to the high value of the refraction index in the longer wave lengths and when this index decreases, the increasing value of the absorption index maintains the high reflectivity. This rise in the absorption index and fall in the refractive index indicates an optical condition in the crystal very similar to

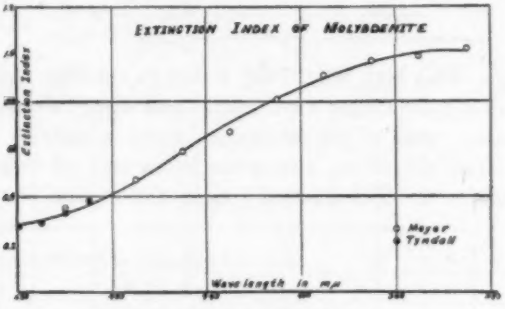


Refraction index of molybdenite.

metals in the ultraviolet, while in the visible and more so in the red, the crystal behaves as a transparent crystal. When crystal surfaces of molybdenite are set so as to produce multiple reflection the crystal yas a steel blue tint. This is a verification of the high reflectivity of the substance in the shorter wave lengths as compared to the longer wave lengths. The curves give the results of Tyndall's investigation and also those of Coblentz. The agreement is very satisfactory. The methods are quite different and some variation might be expected in the different

specimens used. Dr. Coblentz kindly loaned some of the specimens upon which his observations were made and although some time over a year had elapsed no observable variation from the results gained from fresh specimens could be found.

The complicated mathematical relations between experimental observations and final results preclude any accurate determination of the error in the results, but it has been estimated that the error is less than 10 per cent and is much less in the center of the range investigated than at either end.



Extinction index of molybdenite

In conclusion, I wish to thank Dr. E. P. T. Tyndall and other members of the staff of the Physics Department for their interest and assistance, and also to thank Dr. Coblentz of the Bureau of Standards for the loan of his molybdenite specimens.

UNIVERSITY OF IOWA,
IOWA CITY, IOWA,
MARCH, 1926.

THE LIGHT FLUX DISTRIBUTION OF A SYSTEM OF INTERREFLECTING SURFACES

By Ziro Yamauti

To solve the problem of interior illumination, a number of methods have been proposed from both theoretical and experimental investigations and have played a splendid rôle in the actual practice of the illuminating engineering. As has been already pointed out, the increase in illumination due to the interreflections between different surfaces is surprising, and the distribution of flux utilized is entirely different from that given by the source. The present paper is an introduction to a more complete investigation, theoretical and experimental, of illumination probems.

GENERAL THEORY

Suppose that we have n surfaces of given dimensions, shape and nature of reflection, fixed in position. Let us refer to these surfaces as surface (1), surface (2), and so on. Suppose that from a light source unit flux falls on surface (1) and no flux on the others, producing the final incident flux

$$p_{11}, p_{12}, \cdots, p_{1n},$$

on the n surfaces respectively by interreflection, and when flux[1] F_1 falls on surface (1) and no flux to the others, it produces the final incident flux

$$p_{11}F_1, p_{12}F_1, \cdots, p_{1n}F_1,$$

on the n surfaces respectively by interreflection.

Similarly, if unit flux falls on surface (2) and no flux on the others, giving the final incident flux

$$p_{21}, p_{22}, \cdots, p_{2n},$$

then incident flux F_2 on surface (2) leaving the others unlighted by source, produces the final incident flux

$$p_{21}F_2, p_{22}F_2, \cdots, p_{2n}F_2.$$

In the same way we can calculate the result of F_3 falling on (3), F_4 on (4), and so on.

[1] If the surface itself is luminous, the equivalent initial incident flux must be taken. For example, if its brightness be β and its reflection factor ρ, the equivalent initial illumination is β/ρ.

If we now superpose the solutions we have obtained, we find that the effect of simultaneous initial incident flux $F_1, F_2, \ldots . F_n$ is to produce final incident flux $\mathfrak{F}_1, \mathfrak{F}_2, \ldots . \mathfrak{F}_n$, where

$$\left.\begin{aligned} \mathfrak{F}_1 &= p_{11}F_1 + p_{21}F_2 + \cdots + p_{n1}F_n \\ \mathfrak{F}_2 &= p_{12}F_1 + p_{22}F_2 + \cdots + p_{n2}F_n \end{aligned}\right\} \tag{1}$$

These equations give the final incident flux in terms of the initial incident flux. The coefficients $p_{11}, p_{21}, \ldots .$ do not depend on the incident flux, being purely geometrical quantities, which depend on the reflecting nature, dimensions, shape and position of the different surfaces. These coefficients may be called the "coefficients of final incident flux." These can be determined from experimental results.

If we know the final incident flux $\mathfrak{F}_1, \mathfrak{F}_2, \ldots . \mathfrak{F}_n$, we can obtain the values of the initial flux $F_1, F_2, \ldots . F_n$, by solving the equations (1). We obtain the following system of equations of the form

$$\left.\begin{aligned} F_1 &= q_{11}\mathfrak{F}_1 + q_{21}\mathfrak{F}_2 + \cdots + q_{n1}\mathfrak{F}_n \\ F_2 &= q_{12}\mathfrak{F}_1 + q_{22}\mathfrak{F}_2 + \cdots + q_{n2}\mathfrak{F}_n \end{aligned}\right\} \tag{2}$$

etc. The relations of q's and p's are

$$\frac{q_{11}}{\Delta p_{11}} = \frac{q_{21}}{\Delta p_{12}} = \frac{q_{12}}{\Delta p_{21}} = \cdots = \frac{q_{ij}}{\Delta p_{ji}} = \cdots = \frac{1}{\Delta p} \tag{3}$$

where

$$\Delta p = \begin{vmatrix} p_{11} & p_{21} & \cdots & p_{n1} \\ p_{12} & p_{22} & \cdots & p_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ p_{1n} & p_{2n} & \cdots & p_{nn} \end{vmatrix}$$

and Δp_{ji} is the co-factor of p_{ji} in Δp.

The coefficients $q_{11}, q_{21}, \ldots .$ may be called the "coefficients of initial incident flux."

If the manner of reflection be independent of the direction of incident flux and the following relations be established

$$a_{ij} = \frac{F_{ij}}{F_i} \qquad (i, j = 1, 2, \cdots, n) \tag{4}$$

where F_i is the incident flux on surface (i) and F_{ij} is its part which reaches surface (j) by reflection and not by interreflection, we can calculate the values of p's or q's. The one special case of this nature of surface is that of perfect diffusion. a_{ij} may be called the "coefficients of flux distribution."

Let the absorption factor be a_{i0} and the part of the incident flux which does not reach any surface be $a_{i\infty}$. ($a_{i\infty}$ is nil if some of the surfaces enclose all the other surfaces.) Then we have the following relations

$$\left.\begin{aligned} &a_{10}+a_{11}+a_{12}+\cdots+a_{1n}+a_{1\infty}=1\\ &a_{20}+a_{21}+a_{22}+\cdots+a_{2n}+a_{2\infty}=1\\ &\text{etc.}\end{aligned}\right\} \tag{5}$$

$p_{11}-1$ is the amount of increase of incident flux of (1) by interreflection when the initial incident flux is unit flux. Of unit flux incident on (1), a_{10} is absorbed by the surface, a_{11} reaches the surface (1), a_{12} the surface (2), etc. By interreflection a_{11} gives the flux $a_{11}p_{11}$, a_{12} gives $a_{12}p_{21}$, etc. The result of these fluxes is but the final increase of incident flux on (1) and so we have

$$p_{11}-1=a_{11}p_{11}+a_{12}p_{21}+\cdots+a_{1n}p_{n1}$$

or

$$(1-a_{11})p_{11}-a_{12}p_{21}-\cdots-a_{1n}p_{n1}=1\,.$$

p_{21} is the incident flux on (1) by interreflection when unit flux falls on (2). Of unit flux incident on (2), a_{20} is absorbed by the surface, a_{21} reaches the surface (1), a_{22} to (2), etc. a_{21} coming back by interreflection gives the flux $a_{21}p_{11}$ to (1), a_{22} gives $a_{22}p_{21}$, and so on. The result of these fluxes is nothing but the final incident flux on (1) and we have

$$p_{21}=a_{21}p_{11}+a_{22}p_{21}+a_{23}p_{31}+\cdots+a_{2n}p_{n1}$$

or

$$-a_{21}p_{11}+(1-a_{22})p_{21}-a_{23}p_{31}-\cdots-a_{2n}p_{n1}=0\,.$$

In like manner we have the following n groups of n simultaneous equations of p's.

$$\left.\begin{aligned} &(1-a_{11})p_{11} - a_{12}p_{21} - a_{13}p_{31} - \cdots - a_{1n}p_{n1} = 1\\ &-a_{21}p_{11} + (1-a_{22})p_{21} - a_{23}p_{31} - \cdots - a_{2n}p_{n1} = 0\\ &-a_{31}p_{11} - a_{32}p_{21} + (1-a_{33})p_{31} - \cdots - a_{3n}p_{n1} = 0\\ &\text{etc.}\end{aligned}\right\} \quad (6_1),$$

$$\left.\begin{aligned} &(1-a_{11})p_{12} - a_{12}p_{22} - a_{13}p_{32} - \cdots - a_{1n}p_{n2} = 0\\ &-a_{21}p_{12} + (1-a_{22})p_{22} - a_{23}p_{32} - \cdots - a_{2n}p_{n2} = 1\\ &-a_{31}p_{12} - a_{32}p_{22} + (1-a_{33})p_{32} - \cdots - a_{3n}p_{n3} = 0\\ &\text{etc.}\end{aligned}\right\} \quad (6_2),$$

$$\left.\begin{aligned} &(1-a_{11})p_{13} - a_{12}p_{23} - a_{13}p_{33} - \cdots - a_{1n}p_{n3} = 0\\ &-a_{21}p_{13} + (1-a_{22})p_{23} - a_{23}p_{33} - \cdots - a_{2n}p_{n3} = 0\\ &-a_{31}p_{13} - a_{32}p_{23} + (1-a_{33})p_{33} - \cdots - a_{3n}p_{n3} = 1\\ &\text{etc.}\\ &\text{etc. etc.}\end{aligned}\right\} \quad (6_3),$$

Let

$$\Delta a = \begin{vmatrix} 1-a_{11} & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & 1-a_{22} & \cdots & -a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ -a_{n1} & -a_{n2} & \cdots & -a_{nn} \end{vmatrix},$$

co-factor of $-a_{ij}$ be Δa_{ij}, and that of $(1-a_{ii})$ be Δa_{ii}.

Then p's are obtained by the relations

$$\frac{p_{11}}{\Delta a_{11}} = \frac{p_{21}}{\Delta a_{12}} = \cdots = \frac{p_{ij}}{\Delta a_{ji}} = \cdots = \frac{1}{\Delta a} \tag{7}$$

ENCLOSED SURFACE

If the system is enclosed by a surface, we have $a_{i\infty}=0$ and

$$\sum_{j=0}^{n} a_{ij} = 1 \qquad (i=1,2,\cdots,n) \tag{8}$$

The absorbed flux by surface (i) must be $a_{i0}\mathfrak{F}_i$ and sum of the absorbed fluxes all over the surfaces of the system is

$$\begin{aligned} \sum_{i=1}^{n} a_{i0}\ \mathfrak{F}_i &= \sum_{i=1}^{n}(1-a_{i1}-a_{i2}-\cdots-a_{in})(p_{1i}F_1+p_{2i}F_2+\cdots+p_{ni}F_n) \\ &= \sum_{j=1}^{n} F_j \sum_{i=1}^{n}(-a_{i1}-a_{i2}-\cdots+(1-a_{ii})-a_{i,i+1}-\cdots-a_{in})\rho_{ji} \\ &= \sum_{j=1}^{n} F_j \end{aligned}$$

This last expression is nothing but the flux emitted by the sources. Thus the absorption-of-light method proposed by McAllister, that the sum of the fluxes absorbed by the surfaces is equal to the flux emitted by the sources, can be proved using the coefficients of final incident flux and flux distribution.

PERFECT DIFFUSED SURFACE

If the surfaces are of perfect diffusion, it is very interesting to go into details of the nature of these coefficients.

Suppose that the dimensions of the surfaces are so small that both the initial illumination and illumination due to reflected flux may be considered uniform over each surface. (If the surface is too large to be considered as above or its entire surface cannot be seen from other surfaces, divide it into such elementary surfaces as can be considered in the manner above mentioned. These are to be dealt with later.)

Let the initial illumination be E_i and the area S_i. Then the initial incident flux on surface (i) is

$$F_i = E_i S_i .$$

The amount of flux coming to surface (j) by first reflection of (i) is

$$F_{ij} = \int_{S_i}\int_{S_j} \frac{\rho_i E_i dS_i dS_j \cos\alpha_i \cos\alpha_j}{\pi r^2}$$

$$= \frac{\rho_i E_i}{\pi} \int_{S_i}\int_{S_j} \frac{dS_i dS_j \cos\alpha_i \cos\alpha_j}{\pi r^2},$$

where r is the distance from dS_i to dS_j, ρ_i is the reflection factor of (i), α_i and α_j are the emitting and incident angle of the ray on the surfaces dS_i and dS_j. Thus we have

$$a_{ij} = \frac{F_{ij}}{F_i} = \frac{\rho_i}{\pi S_i} \int_{S_i}\int_{S_j} \frac{dS_i dS_j \cos\alpha_i \cos\alpha_j}{r^2} \tag{9}$$

Now put

$$a'_{ij} = \frac{S_i}{\rho_i} a_{ij}$$

which may be called the "coefficient of flux distribution for brightness" because the value of the brightness of (i) multiplied by this coefficient a'_{ij} gives the incident flux on (j) F_{ij}. The value of a'_{ij} is

$$a'_{ij} = \frac{1}{\pi} \int_{S_i}\int_{S_j} \frac{dS_i dS_j \cos\alpha_i \cos\alpha_j}{r^2} \tag{10}$$

and from symmetry a'_{ji} is

$$a'_{ij} = \frac{1}{\pi} \int_{S_j}\int_{S_i} \frac{dS_j dS_i \cos\alpha_j \cos\alpha_i}{r^2} .$$

Thus we have

$$a'_{ij} = a'_{ji} \tag{11}$$

Next put

$$p'_{ij} = \frac{\rho_j}{S_j} p_{ij}$$

which may be called the "coefficient of final brightness" as p'_{ij} is the value of final brightness produced on (j) by unit flux incident on (i). Then $a'_{ij}p'_{ji}$ becomes the final brightness produced on (i) by the flux

a'_{ij} on (j) which is due to the flux on (i) whose brightness is unity, or by the above relations we have

$$a'_{ij}p'_{ji}=\frac{S_i}{\rho_i}a_{ij}\frac{\rho_i}{S_i}p_{ji}=a_{ij}p_{ji},$$

that is, it is equal to the incident flux on (i) by interreflection due to the incident flux a'_{ij} on (j) which is due to first reflection of unit flux initially incident on (i). This relation gives the transformation of the first equation of (61), the second equation of (62), third one of (63), etc., in the following forms

$$\left.\begin{aligned}\left(\frac{S_1}{\rho_1}-a'_{11}\right)p'_{11}-\quad a'_{12}\;\;p'_{21}-a'_{13}p'_{31}-\cdots-a'_{1n}p'_{1n}&=1\\ -a'_{21}\;\;p'_{11}+\left(\frac{S_2}{\rho_2}-a'_{22}\right)p'_{21}-a'_{23}p'_{31}-\cdots-a'_{2n}p'_{n1}&=1\end{aligned}\right\}\qquad(12)$$

etc.

Again we have the brightness produced on (j) by a'_{ik} on (k) which is due to the incident flux on (i) whose brightness is unity.

$$a'_{ik}\,p'_{kj}=\frac{S_i}{\rho_i}a_{ik}\frac{\rho_j}{S_i}p_{kj}=\frac{S_i\rho_j}{S_j\rho_i}a_{ik}\,p_{kj}$$

which transforms the other equations of (6_1) to (6_n).

$$\left.\begin{aligned}-a'_{21}p'_{11}+\left(\frac{S_2}{\rho_2}-a'_{22}\right)p'_{21}\quad-a'_{23}\;\;p'_{31}-\cdots-a'_{2n}p'_{n1}&=0\\ -a'_{31}p'_{11}\quad-a'_{32}\;p'_{21}+\left(\frac{S_3}{\rho_3}-a'_{33}\right)p'_{31}-\cdots-a'_{3n}p'_{n1}&=0\end{aligned}\right\}\qquad(12')$$

etc. etc.

Thus we get the equations of $a'p'$ similar to those of ap given by (6_1) to (6_n) and we must have the similar solutions

$$\frac{p'_{11}}{\Delta a'_{11}}=\frac{p'_{12}}{\Delta a'_{12}}=\cdots=\frac{1}{\Delta a'},\qquad(13)$$

where

$$\Delta a'=\begin{vmatrix}\frac{S_1}{\rho_1}-a'_{11} & -a'_{12}\cdots & -a'_{1n}\\ -a'_{21}\;\frac{S_2}{\rho^2}-a'_{22}\cdots & -a'_{2n}\\ \cdots\cdots\cdots & \\ -a'_{n1} & -a'_{n2}\cdots\frac{S_n}{\rho_n}-a'_{nn}\end{vmatrix},$$

$\Delta a'_{ij}$ is the co-factor of a'_{ij} and $\Delta a'_{ii}$ is that of $(S_i/p_i - a'_{ii})$.
The value of $\Delta a'$ and the co-factors have the following bearing to those of Δa and the corresponding co-factors.

$$\left.\begin{aligned} \Delta a' &= \frac{S_1}{\rho_1}\cdot\frac{S_2}{\rho_2}\cdots\frac{S_n}{\rho_n}\Delta a \\ \Delta a'_{ij} &= \frac{S_1}{\rho_1}\cdot\frac{S_2}{\rho_2}\cdots\frac{S_{j-1}}{\rho_{j-1}}\cdot\frac{S_{j+1}}{\rho_{j+1}}\cdots\frac{S_n}{\rho_n}\cdot\Delta a_{ij} \end{aligned}\right\} \tag{14}$$

Using the coefficients of final brightness we can obtain the value of final brightness B knowing the initial incident flux $F_1, F_2, \ldots . F_n$ by the equations

$$\left.\begin{aligned} B_1 &= p'_{11}F_1 + p'_{21}F_2 + \cdots \\ B_2 &= p'_{12}F_1 + p'_{22}F_2 + \cdots \\ &\text{etc.} \end{aligned}\right\} \tag{15}$$

where B_i is the value of final brightness of surface (i).

As we have by (11)

$$a'_{ij} = a'_{ji},$$

we must have

$$\Delta a'_{ij} = \Delta a'_{ji}.$$

From this or by (12) it follows

$$p'_{ij} = p'_{ji} \tag{16}$$

(16) states that the final brightness of a surface due to the initial incident flux on another is equal to the final brightness of the latter surface due to the initial incident flux on the former, the initial incident flux being the same in each case.

By definition or by (14) we have

$$p'_{ij} = \frac{\rho_j}{S_j}p_{ij}$$

and similarly

$$p'_{ji} = \frac{\rho_i}{S_i}p_{ji}$$

from which we have

$$\frac{\rho_j}{S_j}p_{ij} = \frac{\rho_i}{S_i}p_{ji}$$

or

$$\frac{S_i}{\rho_i}p_{ij} = \frac{S_j}{\rho_j}p_{ji} \tag{17}$$

The last relation shows that the final incident flux on a surface due to the flux on another surface is equal to the final incident flux on the latter surface due to the flux on the former, the initial brightness being the same in each case.

The solutions of (15) give the following relations

$$\left.\begin{aligned} F_1 &= q'_{11}B_1 + q'_{21}B_2 + \cdots \\ F_2 &= q'_{12}B_1 + q'_{22}B_2 + \cdots \\ &\text{etc.} \end{aligned}\right\} \tag{18}$$

where

$$\frac{q'_{11}}{\Delta p'_{11}} = \frac{q'_{21}}{\Delta p'_{12}} = \cdots = \frac{1}{\Delta p'} \tag{19}$$

and

$$\Delta p' = \begin{vmatrix} p'_{11} & p'_{21} & \cdots & p'_{n1} \\ p'_{12} & p'_{22} & \cdots & p'_{n2} \\ \cdot & \cdot & \cdot & \cdot \\ p'_{1n} & p'_{2n} & \cdots & p'_{nn} \end{vmatrix}$$

and Δp_{ij} is the co-factor of p'_{ij} in $\Delta' p$.

The q''s may be called the "coefficients of initial incident flux for brightness."

By (16) we have

$$p'_{ij} = p'_{ji}$$

which gives

$$q'_{ij} = q'_{ji} \tag{20}$$

(20) states that the initial incident flux on a surface necessary to produce a certain amount of final brightness on another surface is equal to the initial incident flux on the latter necessary to produce the same amount of final brightness on the former.

We have

$$\left.\begin{aligned} \Delta p' &= \frac{\rho_1}{S_1} \cdot \frac{\rho_2}{S_2} \cdots \frac{\rho_n}{S_n} \Delta p \\ \text{and} \quad \Delta p'_{ij} &= \frac{\rho_1}{S_1} \cdots \frac{\rho_{j-1}}{S_{j-1}} \cdot \frac{\rho_{j+1}}{S_{j+1}} \cdots \frac{\rho_n}{S_n} \Delta p_{ij} \end{aligned}\right\} \tag{21}$$

(21) gives

$$q'_{ij} = \frac{\Delta p'_{ji}}{\Delta p'} = \frac{S_i}{\rho_i} q_{ij}$$

and hence we have

$$\frac{S_i}{\rho_i} q_{ij} = \frac{S_j}{\rho_j} q_{ji}$$

or

$$\frac{\rho_j}{S_j} q_{ij} = \frac{\rho_i}{S_i} q_{ji} \tag{22}$$

which shows that the initial brightness on a surface necessary to produce final incident flux on another surface is equal to the initial brightness on the latter surface necessary to produce equal final incident flux on the former.

RECIPROCATION THEOREM

If the initial incident flux $F_1, F_2, \ldots$ on the surfaces of a system produce final brightness $B_1, B_2, \ldots$ and if the initial incident flux $F_1', F_2', \ldots$ produce final brightness $B_1', B_2', \ldots$ then

$$\Sigma F B' = \Sigma F' B \tag{23}$$

the summation extending in each case over all the surfaces.

To see the truth of this theorem, we need only notice that, for the surface (i) we have

$$F_i B_i' = F_i (p_{1i}' F_1' + p_{2i}' F_2' + \cdots + p_{ni}' F_n')$$

so that in $\Sigma F B'$ the coefficient of $F_i F_j'$ is p'_{ji}. From symmetry the coefficient of $F_i F_j'$ in $\Sigma F B$ is p_{ij} which is equal to p'_{ij} by (16).

Thus

$$\Sigma F B' = \Sigma F' B .$$

SYSTEM OF LARGE DIFFUSED SURFACES

As already pointed out, the above consideration is based upon the assumption that the surfaces are so small that the initial illumination and illumination due to reflected flux are considered uniform over each surface.

In case of the large surfaces we must divide them into small elementary surfaces and we have from the equations (61) to (6n)

$$\left.\begin{aligned} p(x,x) &= 1 + \sum_s e(x,z)\, p(z,x)\, \delta S_s \\ p(x,y) &= \sum_s e(x,z)\, p(z,y)\, \delta S_s \end{aligned}\right\} \tag{24}$$

where $p(x,y)$ is the coefficient of final incident flux, $e(x,z)$ is given by

$$e(x,z) = \frac{a(x,z)}{\delta S_z} = \frac{\rho_z \cos\alpha_x \cos\alpha_z}{\pi r_{xz}^2},$$

which may be called "the coefficient of illumination" and is nil if x and z do not face each other and the summations are to be extended over the whole space.

Solving the equations (24) we get

$$\left.\begin{aligned} p(x,x) &= 1+\frac{P(x,x)\delta S_x}{P} \\ p(x,y) &= \frac{P(x,y)\delta S_y}{P} \end{aligned}\right\} \tag{25}$$

where

$$P=1-\sum_S e(x,x)\,\delta S_z+\frac{1}{2!}\sum_S\sum_S \begin{vmatrix} e(x,x) & e(x,y) \\ e(y,x) & e(y,y) \end{vmatrix} \delta S_x\,\delta S_y-\cdots$$

and

$$P(x,y)=e(x,y)-\sum_S \begin{vmatrix} e(x,y) & e(z,z) \\ e(z,y) & e(z,z) \end{vmatrix} \delta S_z$$

$$+\frac{1}{2!}\sum_S\sum_S \begin{vmatrix} e(x,y) & e(x,z) & e(x,u) \\ e(z,y) & e(z,z) & e(z,u) \\ e(u,y) & e(u,z) & e(u,u) \end{vmatrix} \delta S_z\,\delta S_u-\cdots$$

Let the initial and final illumination on (x) be $E(x)$, and $\epsilon(x)$. We have from (1)

$$\epsilon(x)\delta S_x=\sum_S p(y,x)E(y)\delta S_y$$

or

$$\epsilon(x)=\sum_S \frac{p(y,x)}{\delta S_x}E(y)\delta S_y$$

$$=E(x)+\frac{1}{P}\sum_S P(y,x)E(y)\delta S_y.$$

Making $\delta S_y\to 0$, and writing the summations as integrations, we are led to consider the possibility of the equation

$$\epsilon(x)=E(x)+\frac{1}{P}\int_S P(y,x)E(y)\,dS_y$$

where

$$P=1-\int_S e(x,x)\,dS_x+\frac{1}{2!}\int_S\int_S \begin{vmatrix} e(x,x) & e(x,y) \\ e(y,x) & e(y,y) \end{vmatrix} dS_x\,dS_y-\cdots$$

and

$$P(x,y)=e(x,y)-\int_S \begin{vmatrix} e(x,y) & e(x,z) \\ e(z,y) & e(z,z) \end{vmatrix} dS_z$$

$$+\frac{1}{2!}\int_S\int_S \begin{vmatrix} e(x,y) & e(x,z) & e(x,u) \\ e(z,y) & e(z,z) & e(z,u) \\ e(u,y) & e(u,z) & e(u,u) \end{vmatrix} dS_z dS_u - \cdots$$

SPECIAL CASE

As an example of the above method of treatment let us obtain the illumination on the wall of a hollow spherical cap of perfect diffusing surface without any foreign bodies near it. In this case the coefficient of illumination is equal to $\rho/\pi D^2$ all over the surface where ρ is the reflection factor and D is the diameter of the sphere. Let the area of the cap be $\pi D^2\omega$. We have

$$P = 1-\rho\omega$$

$$P(x,y) = \frac{\rho}{\pi D^2}$$

so that

$$e(x) = E(x) + \frac{\frac{\rho}{\pi D^2}}{1-\rho\omega}\int_S E(y)dS_y$$

$$= E(x) + \frac{\rho\omega E}{1-\rho\omega}$$

where E is the average initial illumination of the cap.

For the complete sphere $\omega = 1$ and we get

$$e(x) = E(x) + \frac{\rho E}{1-\rho} = E(x) + \frac{\rho F_t}{\pi D^2(1-\rho)}$$

where F_t is the total flux emitted by the source. This expression is the same as that obtained by other methods.

ELECTROTECHNICAL LABORATORY,
MINISTRY OF COMMUNICATIONS,
TOKYO, JAPAN
JANUARY 9, 1926.

Cathode Ray Oscillograph for Several Simultaneous Waves with Stabilized Linear Time Axis. A linear time axis for cathode ray oscillograph is obtained by repeatedly charging a condenser through a saturated thermionic rectifier and discharging through a neon lamp when a critical voltage is reached; Bailey, Phys. Rev., *25*, p. 585; April, 1925. The period (controlled by condenser capacity and rectifier filament current) can be made approximately synchronous

with periodic phenomena under study, so that the oscillograph shows a nearly stationary wave in rectangular coordinates. This has proved very satisfactory for graphic demonstration of heart sounds, apparatus for which, developed by the Western Electric Co., was exhibited at the 1925 meetings of Amer. Med. Assoc. and A. A. A. S.

For certain alternating current study, precise synchronism can be obtained by means of a synchronus contactor, a method found satisfactory, but limited in its application and involving unnecessary auxiliary equipment. The result is simply and satisfactorily obtained by introducing some of the alternating current into the neon circuit, thus synchronizing the discharge of the condenser, withut affecting its uniform rate of charge. This gives an absolutely stationary alternating current wave, irrespective of its source or frequency, which may be copied on tracing cloth or by binocular vision. By persistence of vision two or more waves can be readily obtained simultaneously by a motor-driven switch making connection to the several sources rapidly in succession.—[F. Bedell and H. J. Reich, Cornell University; Science, *63*, p. 619, 1926.]

Aircraft Instruments. H. N. Eaton, K. H. Beij, W. G. Brombacher, W. W. Frymoyer, H. B. Henrickson, C. L. Seward, and D. H. Strother, all of the Aeronautic Instrument Section, U. S. Bureau of Standards. xii + 269 pp. 10 plates and 58 line engravings. Published by The Ronald Press Company, New York, 1926. $5.00.

This most recent volume of the Ronald Aeronautic Library describes the mechanism and outlines the principles of operation of practically every kind of aircraft instrument in use to-day. Over 175 types and makes of instruments are included, the book being the most complete of its kind in the English language. Instruments of European, as well as American, manufacture are described by the authors under the following chapter headings: Altitude and Climb Measurements; Air and Ground Speed Measurement; Direction and Position Instruments; Power Plant Instruments; Manometers; Special Instruments for Airships; Instruments for Measuring Gas and Air Temperatures; Navigating Instruments; Oxygen Supply Instruments; Special Test Instruments. Conversion factors and performance specifications for a number of the more common service instruments are given in two appendices. Some of these specifications may be applied to instruments used in other technical fields, as in the storage of gases and liquids, and in automobiling.

References to original sources are freely given and form a valuable feature of the book; thus, in the chapter on Air and Ground Speed Measurement, sixty nine such references are given. Although intended primarily for those interested in a special field, the book has much material—the description of the sun compass used by Amundsen, for example—of interest to the casual reader. The book is well prepared and is authoritative. FREDERICK BEDELL.

NOTE ON THE RARE EARTHS AS ACTIVATORS OF LUMINESCENCE*

By E. L. Nichols and H. L. Howes

The observations to be recorded in this note were made upon salts of samarium, europium, thulium, dysprosium, terbium, and of neodymium, praesodymium, yttrium and erbium, prepared for us by Professor C. James. Traces of these elements (whatever be the combination in which they occur as the result of the various heat treatments applied) were put into solid solution in various solvents i. e.

(1) CaF_2 or CaO.

(2) Borax glass (from borax).
$NaPO_3$ (from microcosmic salt), NaF.

(3) Al_2O_3.

The three solvents mentioned under (2) were used in the form of "beads" on platinum wire into which the activator could be introduced readily in the desired amounts. The attempts to activate such beads by means of the rare earths were suggested by the interesting results obtained in a recent study of *uranium as an activator.*[1] Since most of the elements mentioned above have been studied in considerable detail by Crookes, Le Coc de Boisbaudran, Urbain and others there will be given here only such features of our observations as are thought to be new or to afford confirmation or suggest modification of the conclusions of the earlier workers in this field.

The luminescence spectra were observed and roughly mapped by means of a hand spectroscope with direct reading wave-length scales. All finer details, for the study of which this instrument was inadequate were left for subsequent study. It may be remarked however in this connection that except for the accurate location of very narrow line-like bands and the resolution of close groups an optically good instrument of small dispersion has many advantages, especially in a rapid preliminary survey of new spectra. Excitation to luminescence was by the iron spark and by cathode-bombardment respectively.

* Received June 30, 1926.

[1] Nichols and Slattery: *12*, p. 449; *1926.* J.O.S.A. & R. S. I.

LUMINESCENCE OF THE VARIOUS COMBINATIONS

In Table 1 we have recorded for comparison the effects produced by impregnating with traces of various rare earths the solid solvents already enumerated.

TABLE 1 (a). *Effect of the rare earth elements as activators.*

Activator	Solvent	Luminescence	
		Photo	Cathodo
Thulium	$NaPO_3$	None	None
	NaF	None	red (broad bands)
	Borax	pale apple green (narrow bands)	violet (narrow bands)
	CaF_2	inert	bright; numerous line-like bands (see Table 2).
	CaO	inert	fine blue; (many line-like bands).
	Al_2O_3	inert	blue v.d. or after heating to 900°C dim white-continuous spectrum.
Europium	$NaPO_3$	pink; v.d.	red; narrow bands (see Table 3).
	NaF	inert	red; continuous spectrum.
	Borax	cherry red; (bright)	ruddy; bands at .618, 625μ
	CaF_2	ruddy; line-like bands	ruddy; very complex (see Table 4).
	CaO	ruddy; bands very bright, line-like	ruddy; bands very bright; line-like
	Al_2O_3	line-like v. d.	line-like (see Table 5).
Dysprosium	$NaPO_3$	opalescent	inert
	NaF	inert	orange; bright bands on continuous background. .556, 540 .480.μ
	Borax	green; (line-like bands)	green; line-like bands
	CaF_2	very bright; line-like bands	very bright; line-like bands
	CaO	very bright; line-like bands	very bright; line-like bands
Terbium	$NaPO_3$	inert	inert
	Borax	green; very bright	green; very bright
	CaF_2	———	yellow-green; bright line-like bands.
	CaO	———	yellow green; bright line-like bands.
Samarium	$NaPO_3$	red; narrow bands at .647, 602, 542μ	red; narrow bands at .647, 602, 542μ
	NaF	inert	red; very dim
	Borax	red; (see Table 7)	red; (see Table 7)
	CaF_2	red	very bright; (see Table 7)
	CaO	red	very bright; (see Table 7)

TABLE 1 (b). *Effect of activators (continued).*

Activator	Solvent	Luminescence	
		Photo	Cathodo
Yttrium	$NaPO_3$	green blue (dim)	inert
	NaF	inert	white; continuous spectrum.
	Borax	inert	white; very dim.
	CaF_2	purple	ruddy; broad bands in red, green, violet
	CaO	inert	blue; terbium bands on continuous background
Neodymium	$NaPO_3$	inert	inert
	NaF	blue	red (continuous spectrum.)
	Borax	inert	inert
	CaF_2	inert	shows samarium
	CaO	not recorded	bands not recorded
Praseodymium	Borax	inert	inert
	CaF_2	———	orange-red, very bright. Spectrum highly resolved with many line-like bands.
Erbium	Borax	green (terbium lines)	Very bim.
	NaF	green (continuous sp.)	
	CaF_2	———	Terbium lines (no continuous spectrum.

DISCUSSION

(1) *Superiority of* CaO *as a solvent.* In the case of the elements listed in Table 1a solutions in calcium oxide were found to be more brilliantly luminescent that where the other solvents mentioned above were employed, although CaF_2, the activity of which may be due to superficial conversion of the surface into the oxide, was almost as good. Our solutions in CaO were made by dissolving the rare earth salt in nitric acid, diluting the solution, moistening powdered calcium carbonate with a sufficient quantity of the solution to give approximately the optimum proportions (say two tenths per cent) and treating in a porcelain crucible to redness (about 700°). Thus prepared they were, in general, exceedingly brilliant in the cathode tube and gave spectra in which the bands were much more nearly line-like and better defined than those depicted by Urbain and the other earlier observers. These spectra are indeed only to be compared in beauty with those recently

obtained by Miss F. G. Wick[2] who heated to incipient fusion fluorspars containing samarium and other of the elements of that group.

Whether the quality of the spectra thus obtained is to be ascribed to the unusual purity of the activating elements, must for the present be regarded only as a surmise.

DOMINANCE AND SUPPRESSION

The tendency recorded by earlier writers for certain activators to suppress the luminescence due to other elements present in the solvent was strikingly confirmed by some of our observations.

For example the calcium oxide used in our preparations glows strongly under cathodo excitation with the characteristic red-yellow color due to the presence of manganese; as seen in the calcites from Franklin Furnace and from many other localities.[3] The spectrum of this glow, as is well-known is so broad banded as to be practically continuous. When any one of the rare earths mentioned in Table 1 (a) was added and the mixture was treated as above described the continuous spectrum disappeared almost completely leaving only the line-like bands produced by the activating action of the rare earth.

A striking instance of this selective activation was observed when neodymium was introduced into CaO. The spectrum invariably obtained was that of samarium.

In the separation of praesodymium from neodymium, as is well known, samarium tends to go into the same fraction as the latter element. It is therefore the impurity chiefly to be expected in samples of neodymium. The neodymium at our disposal was exceptionally pure so that the solid solution of neodymium in calcium oxide probably contained only a few parts in a hundred thousand of samarium; a quantity sufficient however to completely suppress the luminescence due to all other possible activators which may have been present.

It is probable that the amount of praseodymium present in such a preparation under the conditions described would exceed that of samarium but the line-like and characteristic praesodymium spectrum was not to be seen. On the other hand our samples of praesodymium doubtless contained traces of both neodymium and samarium but the luminescence of both these activators was suppressed. It would appear therefore: (1) that the luminescence of neodymium is suppressed by minute traces of either samarium or praesodymium; (2) that traces of

[2] Frances F. Wick. Physical Rev. (2) *24*, p. 272; 1924.

[3] Nichols, Howes and Wilber; Physical Rev. (*2*) *12*, p. 351; 1918.

samarium will suppress the luminescence due to traces of praseodymium when the dilution is of the same order but that where praseodymium greatly preponderates the samarium bands are suppressed.

Yttrium and erbium afford other interesting illustrations of dominance and suppression.

When yttrium is in solid solution in CaF_2 the line-like luminescence bands of terbium, which is present as a minute impurity, are suppressed whereas in CaO the terbium bands dominate. On the contrary the luminescence of erbium in CaF_2 is suppressed and the terbium spectrum comes out strongly. A comparison of the cathodo-luminescence of our preparations of terbium and of dysprosium in CaO was also instructive in this connection. The dilution of these preparations was approximately equal, i.e. .002 : 1. And they were prepared in a similar manner.

The luminescence of dysprosium contains a prominent band in the red (.675μ) whereas terbium has no bands of wave lengths longer than the group at .625−.627. The dysprosium spectrum on the other hand, according to the measurements of Urbain,[4] terminates towards the shorter wave lengths with the group at .565−.595, while the spectrum of terbium extends throughout the blue and violet.

In the case of our preparations however the dysprosium spectrum, in addition to the dysprosium bands contained the terbium bands also, including those of shorter wave lengths. The terbium spectrum on the other hand was devoid of dysprosium bands.

This might be taken to indicate that the terbium was quite free from dysprosium, which was unlikely, or that a minute trace of terbium in the presence of a relatively much larger amount of dysprosium is capable of giving its characteristic spectrum whereas in the presence of larger amounts of terbium dysprosium is inert.

In a word in a solution where terbium predominates the dysprosium bands disappear; in a solution in which dysprosium predominates both spectra appear.

A description of the luminescence of terbium follows in the section on spectral structure.

PERSISTENT PHOSPHORESCENCE AS A CRITERION

Under the conditions of our experiments the following very simple criterion for terbium in the presence of dysprosium was discovered. We noticed that after the close of excitation the bands of the terbium spectrum persisted whereas those due to dysprosium vanished at once.

[4] Urbain: Ann. de Chimie et de Physique, *8*, 18, p. 222; 1909.

In the after glow the spectrum of dysprosium is therefore absent leaving that of terbium behind. In the intervening regions in the green and yellow of the spectrum where the groups of bands due to these two activators over-lap, this affords a convenient means of identification which is applicable to all the components which are strong enough to be seen in the phosphorescent after-glow.

SPECTRAL STRUCTURE

(1) THULIUM

(a) In NaF with a sufficient admixture to give the bead a very faint yellow body color the spectrum under cathode bombardment showed two broad over-lapping bands the location of which was approximately as follows.

	edge	crest	min.	crest	edge
λ	.555μ	.575μ	.587μ	.610μ	.655μ
$10^4/\lambda$	———	1739(A)*	———	1629(C)*	———

* Letters A and C refer to the relation of these frequencies to the sets thus designated in Table 2, to follow.

After bombardment the bead was found to have acquired a pale pink body color.

It was observed that beads of NaF containing thulium were clouded when cold, became clear when fused in the hydrogen flame and at a much higher temperature clouded up again and gave off an intense white light.

The discussion of the visible emission accompanying this and other transformations occurring in these beads is reserved for a separate communication.

(b) In CaF_2. To a sample of the fluoride (from Kahlbaum) which when fused to a bead without admixture had, after cooling, a bright cathodo-luminescence with continuous spectrum, a trace of thulium oxide was introduced.

The continuous spectrum was completely suppressed, as in the examples mentioned on a previous page, and a spectrum consisting of numerous line-like bands appeared. The detailed study of this spectrum will be undertaken later together with other of the rare earth spectra. A preliminary survey with the hand spectroscope gave the following approximate locations for the more conspicuous bands.

TABLE 2. *Bands in the spectrum of thullium in solid solution in* CaF_2

Set *A*			Sets No. *B*, *C* and *D*			
μ	$10^3/\mu$	Δ		μ	$10^3/\mu$	Δ
.660	1515					
		111 = 7×16−1				
.615	1626		*B*	.520	1923	
		98 = 6×16+2				
.580	1724					
		49 = 3×16+1				
.564	1773		*C*	.455	2198	
		64 = 4×16±0				64=4×16±0
.544	1837			.442	2262	
		32 = 2×16±0				
.535	1869		*D*	+90	2041	
.467	2141					
Interval (from extremes) 16.05						

To designate single bands as members of a *set* of which no other members have been found may seem unwarranted, but our knowledge of such spectra would indicate the improbability of individual bands not thus connected. Band 1923 (set *B*) has in fact been found to belong to a set of emission bands whose relation to the luminescence of thulium is to be considered elsewhere.

(c) In CaO. When prepared as already described the spectrum under cathodo-bombardment consisted of many fine line-like bands. The ruddy fluorescence characteristic of the CaO, and due to traces of manganese was totally suppressed. The spectrum, which was evidently not identical with that obtained when CaF_2 was the solvent, has not yet been mapped.

(d) In Al_2O_3. Using a procedure similar to that already described but with Al_2O_3 (from alum) instead of $CaCO_3$, a very feeble blue cathodo-luminescence was obtained having a spectrum in which the bands were barely discernible. On further heating to about 900° the fluorescence of the cooled powder was white with a continuous spectrum in which the thulium lines were totally suppressed. In this medium it would seem that thulium is at best a very indifferent activator with a tendency to be suppressed instead of becoming dominant. (Compare with the observations on Europium in Al_2O_3, to follow.)

(2) EUROPIUM

(a) In $NaPO_3$, a trace of Europium added to the bead gave a faint pink luminescence. The spectrum was not easily observable.

Under bombardment of cathode rays however there was a ruddy glow and it was possible to note that the spectrum consisted of rather narrow but by no means line-like bands. The crests, very roughly, were located thus:

TABLE 3. *Bands in the spectrum of europium in solid solution in* $NaPO_3$

Brightness	μ	$10^3/\mu$	Remarks
Very very dim	.660	1575	———
Bright	.615	1626	Belongs to set *B* Table 4
Dim	.592	1689	Belongs to set *A* Table 4
Bright	.540	1852	Belongs to set *A* Table 4

(b) In the borax bead europium gave a rich cherry red luminescence, under both photo and cathodo excitation, with banded spectrum. Bands at .625 and .618 (Set *A*, Table 4) were dominant.

(c) In NaF a trace of europium gave a bead clear when fused, clouded when cold. The bead glowed a fine pink color during the transformation. It was inert under the Fe spark but glowed red in the cathode tube. The spectrum showed no narrow bands.

(d) In CaF_2 (Kahlbaum) there was a ruddy fluorescence with line-like bands throughout the spectrum, those in the red being very bright. Cathode bombardment gave a very complex spectrum of which the following components were tentatively located with the hand spetroscope.

TABLE 4. *Bands in the spectrum of europium in solid solution in* CaF_2

Set *A*			Set *B*		
μ	$10^3/\mu$	Δ	μ	$10^3/\mu$	Δ
.625	1600	18	.650	1538	
					180 = 10×18.0
.618	1618	161 = 9×17.9	.582	1718	
					36 = 2×18.0
.562	1779	163 = 9×18.1	.570	1754	
					108 = 6×18.0
.515	1942	107 = 6×17.83	.537	1862	
.487	2049				
Av. interval Set *A* = 17.96			Av. interval set *B* = 18.00		

The following three bands, which are also present, are probably due to samarium.

.590	1695
.550	1818
.526	1901

(e) In CaO (concentration about 1 : 333) after a mild ignition in a quartz crucible, europium gives a ruddy fluorescence both under the iron spark and in the cathode tube. The very brilliant and complex group of exceedingly fine line-like bands were reserved for subsequent detailed study.

(f) In Al_2O_3 (from alum), prepared by inserting a minute quantity of europium oxalate in a cavity in a mass of the Al_2O_3 and heating to incipient fusion; a line-like spectrum comparable to that in CaF_2 but very dim was obtained. The following bands were located (Table 5). The fluorescence line of the ruby which is strong in preparations from this sample of aluminum oxide was very feebly visible as though partly suppressed by the trace of europium. In the consideration of this question the following experiment is instructive.

A splinter from an alundum crucible was dipped into a dilute solution of europium nitrate, and the tip was heated to fusion in an O-H flame. After cooling the splinter was subjected to cathode bombardment. The result was as unexpected as it was striking. The fused tip glowed red and gave the ruby spectrum only. Then followed a dark neutral zone which had not been heated to fusion and was not cathodoluminescent. Beyond this and very clearly defined was a broad region which glowed strongly under bombardment and showed the spectrum given in Table 5. The red ruby line was very very dim.

TABLE 5. *Bands in the spectrum of europium in solid solution in* Al_2O_3

Set *A*			Set *B*		
μ	$10^3/\mu$	Δ	μ	$10^3/\mu$	Δ
			.650	1538	
					54 = 3×18
.551	1815		.628	1592	
		72=4×18			18
.530	1887		.621	1610	
					54 = 3×18
			.601	1664	
					19=18+1
			.594	1683	

The ruby band of fluorescence at 690μ was also very dimly visible.
Both sets *A* and *B* coincide with *A* and *B* in Table 4.

The question of the effect of heat treatment upon the suppression of spectra was further tested by making up samples of europium nitrate in Al_2O_3. These were heated to various temperatures and it was ob-

served that up to the point of fusion brightness and definition increased but that when complete fusion occurred the ruby spectrum appeared and the europium bands vanished.

When however a crystal of fluorspar was dipped in the europium solution (in HNO_3) and one corner was heated to fusion it was in the fused portions that the band spectrum was fully developed and brilliant. Those parts of the crystal which had not been fused glowed but very feebly. Although only one band of those observed in this spectrum and recorded in Table 5 coincides with a band in the spectrum of Eu in CaF_2 the relation between the two is an intimate and remarkable one. As will be seen from the tables the interval 18 is common to both spectra and moreover the same identical sets *A* and *B* occur and to one or the other of these all the bands belong. In one sense therefore the two spectra have an identity.

If we imagine Set *A*, when complete, to consist of components equally spaced and running through the spectrum with a frequency interval of $18\pm$ and Set *B* to consist of a similar succession of components displaced from Set *A* by 7 frequency units, then we can produce the actual spectrum recorded in Table 4 by suppressing all the components in both sets not given in that table and can pass from Table 4 to Table 5 by suppressing components appearing in the former table and substituting those of Table 5. Thus although the spectra may have no bands in common they are parts of the same system. This relationship we may designate as the *Principle of Essential Identity*.

TERBIUM AND SAMARIUM

Table 6 gives the bands due to the luminescence of terbium in three different solvents. The spectrum of this element in CaO and CaF_2 consists of narrow line-like bands. In borax glass where only seven bands were located the resolution is somewhat less complete.

These spectra afford an excellent example of essential identity because of the number of bands common to all three. Set *A'*, which is situated less than three frequency units towards the violet from set *A*, could not be distinguished with certainty from the latter set were the gap much narrower.

TABLE 6. *Bands in the spectrum of terbium in various solid solutions*

	CaO(1)			CaF_2(2)		Borax(3)	
	μ	$10^3/\mu$		$10^3/\mu$		$10^3/\mu$	
				1739			
	.565	1770			31 = 2×16−1		
				1770			
			16				
	.560	1786					
			16		48 = 3×16±0		
	.555	1802					
Set *A*			16				
	.550	1818		1818		550	1818
			143 = 9×16−1		223 = 14×16−1		
	.510	1961		2041			
			96 = 6×16±0		16		
	.486	2057		2057		486	2057
			31 = 2×16−1				
	.479	2088					
		1645					
			64 = 4×16±0				
		1709		1709			
			15 = 16−1				
		1724			319 = 20×16−1		1724
Set *A'*			304 = 19×12±0				
		2028		2028			
			209 = 13×16+1		209 = 13×16+1		
		2237		2237	161 = 10×16+1		
				2398			
		1669		1669			
					49 = 3×16+1		
				1718			
Set *B*			176 = 11×16±0				
					127 = 8×16−1		
		1845		1845		.542	1845
			31 = 2×16−1				
		1876	112 = 7×16±0		143 = 9×16−1		
		1988		1988			
					191 = 12×16−1		
			305 = 19×16+1	2179			
					114 = 7×16+2		
		2293		2293			

(1) Band at .625 due to Eu is omitted.

(2) Bands at .630μ, also .625μ. 594μ and 530μ (Eu) are omitted.

(3) Three other lines in vorax which are displaced two units from Set *A* and are probably composites of Sets *A* and *B* due to incomplete resolutions are located at .622μ .598μ and .494μ.

The luminescence of samarium (Table 7) still further enforces the principle of essential identity. The interval here is again approximately 16 and we find that the main set for the spectrum of the solution in

borax falls in with Set *A* of the spectra of the preparations with CaO and CaF_2.

TABLE 7. *Bands in the luminescence spectrum of samarium*

	Solvent								
	CaO			CaF_2			Borax		
	μ	$10^3/\mu$	Δ	μ	$10^3/\mu$	Δ	μ	$10^3/\mu$	Δ
Set *A*	.580	1724		.580	1724		.640	1563	
			15			15			76
	.575	1739		.575	1739		.575	1739	
			63			63			31
							.565	1770	91
	.555	1802		.555	1802		.510	1961	
Set *B*	.678	1475							
			79						
	.643	1554					.650	1539	
			63						
	.618	1617		.619	1617				
			64			64			
	.595	1681		.595	1681				
			60			31			
	.543	1841		.584	1712				
						65			
				.563	1777				
Set *C*	.605	1653							
			48						
	.588	1701							
			63						
	.567	1764							
			64						
	.547	1828							
Set *C'*	.622	1607		.649	1541				
				.604		114			
					1655		.610	1639	
						76			33
				.546	1831		.598	1672	
Lines due to europium		1587			1587				
		1887						1887	

Similarity between these latter solutions here, as in our previous comparisons, might be ascribed to the formation of traces of CaO in the course of the heat treatment of the CaF_2, but this cannot be true where borax-glass is the solvent and we revert to the proposition

already formulated that the location of the sets depends only upon the activator and is the same for the various solvents used.

Set *A* in the spectra of samarium and of terbium is practically identical as to location, but the grouping of the other sets with reference to it is altogether different as will be seen from the diagram in Fig. 1.

FIG. 1. *Recurring bands in a "full set".*

This represents the grouping of bands which would occur in a region of the spectrum where all the members of the so-called sets were represented by actual bands. The spectrum then, like that of the luminescence spectrum of the uranyl salts, would consist of a system of recurring groups of bands, equidistant and similar.

In the samarium spectrum the grouping is symmetrical about Set *A* with frequency intervals of six units to the *B* and *C* sets respectively. Since the group spacing is 16 this gives only four units between each group of three. With this arrangement nine out of every sixteen bands, provided the distribution were entirely fortuitous, would in general be placed in some set if a departure of one unit on either side of true coincidence were permitted.

In the terbium spectrum the three sets are much more closely grouped and the groups are nearly symmetrical about A' instead of *A* of Fig. 1. Here only seven out of sixteen fortuitously located bands would find a place. The fact that all the observed bands in these spectra have been placed with very few departures greater than one unit quite disposes of the question of chance groupings.

GAPS IN THE SETS

The numerous gaps in sets as compared with the positions occupied by visible bands is accounted for in part at least by the methods commonly employed in studying luminescence spectra. The location of bands by the usual process of inspection and setting to the crests or of photographing the spectrum and locating the visible crests in the

spectrogram, leads to the detection of only a few of the more prominent bands while many overlapping and more or less submerged components cannot be detected. Thus where Urbain recorded 11 bands in the spectrum of samarium in CaO, Tanaka[5] who used the spectrophotometric method previously developed by the present writers[6] listed 70 components within the same spectral region. Similarly Fisher[7] who photographed the seemingly continuous spectrum of columbium in zinc sulphide and explored the spectrograms thus obtained finds seventy-five components which fall into six sets all having the same interval of 21+.

Every luminescent preparation inevitably contains minute traces of various activators perhaps much too minute to be detected by any known method of chemical analysis but quite sufficient to produce luminescence. A. A. Guntz[8] for example, in his very complete investigation of the luminescence of zinc sulphide finds that copper is universally present as an activator in that solvent and that no known method suffices to eliminate it. Similarly it has recently been shown[9] that in NaF uranium produces brilliant luminescence when present in the proportion of $1 : 10^7$ mole fractions. Fortunately *dominance and suppression* operate to render the study of luminescence spectra less complex and the principle here described as *essential identity* brings into a simple system bands which at first appear to be unrelated.

SUMMARY

Various rare earth salts were put into solid solution in CaO, CaF_2, Al_2O_3, $NaPO_3$, NaF and borax glass.

The photo and cathodo luminescence was observed and spectra mapped.

Various cases of the dominance of one activator over others were noted and the conditions studied.

The line-like bands in the spectra of the solid solutions containing traces of thulium, europium, terbium, and samarium were approximately located and arranged in "sets" having a common and constant frequency interval.

[5] Tanaka, J.O.S.A., & R.S.I. *8*, p. 287; 1924.

[6] Nichols, Proc. Am. Philos. Soc., *56*, p. 258; 1917.
Nichols, Howes and Wilber, Physical Review, (*2*), *12*, p. 351; 1918.
Nichols and Howes, Publ. Carnegie Inst. of Washington, *298*; 1919.

[7] Fisher, A photographic study of cathodo luminescence (in progress) in Physical Laboratory of Cornell University.

[8] A. A. Guntz, Thesis on the luminescence of Sidot Blende, Marson; Paris, 1924.

[9] Nichols and Slattery, J.O.S.A. & R.S.I., *loc. cit.*

The frequency interval was found to be 18 (almost exactly) in the case of europium and 16 for the spectra of thulium, terbium and samarium.

The principle of *essential identity* first established for the spectra of solid solutions containing uranium was found to apply to the luminescence of these rare earths : i.e. there are one or more sets of positions in the spectrum which are characteristic of a given activator and whatever the solvent all the luminescence bands of substances containing the activator in question are so located that their crests coincide with these positions.

Spectra thus related are said to be essentially identical although the particular positions in the set thus occupied may not be the same for the different spectra.

CORNELL UNIVERSITY,
ITHACA, NEW YORK,
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Spectrum of Liii.—This, as the spectrum of an atom presumably composed of two electrons and a nucleus (of charge $+3e$), is of unusual interest. It is difficult to produce, perhaps because of the exceptionally high ratio of the second to the first ionizing potential of lithium; it requires about 12 times as much energy to extract two electrons from the Li atom as to extract one. Schüler and Sugiura use a discharge in a tube with hollow cylindrical cathode (Paschen's method) coated with a lithium salt, the Liii lines appearing in the spectrum of the luminous glow within the cylinder; Werner bombards with electrons (of energy ranging up to 1400 equivalent volts) a stream of Li atoms issuing through a small hole in the wall of a crucible, all the apparatus being evacuated. There is agreement in the wave lengths of the principal lines, but grave disagreement as to the assignment of these lines to series, in spite of the assistance which one would expect the helium spectrum to afford. In fact the lines which Morand assigns to a system corresponding to the parhelium spectrum are chiefly assigned by the two others to a system corresponding to the orthohelium spectrum, and vice versa. Sugiura cites some close agreements between frequencies of Liii and quadrupled frequencies of Hei to support his case; they are for lines involving D, d, F or f terms, as one would expect. The data are further complicated by astonishingly rich fine-structures of some of the lines discovered by Schüler, for instance eleven components of the line near 5484A!—[H. Schüler (Potsdam), Naturwiss., July 11, 1924; Ann. d. Phys., *76*, pp. 292–298; 1925. S. Werner (Copenhagen), Nature, *115*, pp. 191–192; 1925. Y. Sugiura (Paris), Jour. de Phys., (6), *6*, 323–333; 1925.]

KARL K. DARROW

The Origin, Nature and Influence of Relativity. By George David Birkhoff, Professor of Mathematics at Harvard University. ix+185 pages, including 21 figures. The Macmillan Company. New York.

Another book on Relativity! The author has written up in book form a series of lectures given partly at the Lowell Institute, partly at the Southern Branch of the University of California. Since (according to the preface) "the theory of Einstein constitutes a revolutionary advance comparable with that due to Copernicus," which is "likely to affect the direction of mathematical, physical and philosophical development" (the reviewer assumes that "moral" and "religious" were inadvertently omitted from the list of qualifying adjectives!) it has been the author's purpose to portray clearly "the origin, nature, and influence of these basic ideas (out of which arises the imposing superstructure of the physical sciences) and their modification in the theory of relativity." In the author's opinion, "the range of material covered is not beyond the comprehension of anyone possessing a modicum of scientific training."

Starting with Euclid and Newton, the concept of Relativity is developed in the first three chapters. One brief chapter, Chapter IV, disposes of the experimental tests of relativity. The remaining four chapters present some consequences of the theory. The book is well written and is as non-mathematical as is consistent with a logical treatment.

But the book suffers from a defect common to most books on Relativity: namely, an almost entire absence of references to original sources, particularly experimental. For example, in the chapter on the experimental tests of Relativity, there is but a single reference; and that to a statement which calls into question Miller's recent experiments at Mount Wilson on the basis of "elusive errors." In connection with the original Michelson-Morley experiment the statement is made that no shift of the interference bands *was ever found*. Whereas Miller's recent experiments are qualitatively in entire accord with the earlier measurements in finding a shift smaller than was expected, *but not zero*. No reference is made to those solar eclipse measurements which are negative so far as confirming relativity is concerned.

We need more books on Relativity. But we need those which will discuss critically the experimental evidence *pro and con*. However, if one's interest lies in the field of abstract mathematical deductions, and the resulting philosophical implication, it is to be doubted whether one can find a better exposition, in brief form, than this one by Dr. Birkhoff.

F. K. RICHTMYER

INSTRUMENT SECTION

ON THE CORRECTION FOR HEAT EXCHANGES BETWEEN A CALORIMETER AND ITS SURROUNDINGS

By Paul E. Klopsteg

One of the best discussions on the subject of the title of this paper appears in the fourth edition (1925) of Ostwald-Luther's "Messungen." The treatment is analytical and an illustration based on an actual experiment indicates how the corrections are to be made. The entire discussion occupies something over eight pages. The student of college physics would probably not have access to this book, and if he had, it would be a task of no small magnitude to apply the discussion to his particular problem. Naturally he would turn to the physics laboratory manuals available in the library for the necessary information.

The writer has examined the sections on heat in most of the commonly used manuals. With one or two exceptions[1] the method of finding the corrected temperature rise in calorimeter experiments has received scant attention from the authors of the manuals. Most of them, prompted by the necessity of conserving space, have so simplified the process they describe that the resulting gain in accuracy is hardly more than theoretical.

Teachers of college physics will agree that any method by which the correction is to be made for exchange of heat between a calorimeter and its surroundings shall conform as closely as possible with theoretical requirements. It should, at the same time, be sufficiently simple in its application for the student to comprehend and apply readily. A rigorous method is of necessity not characterized by elementary simplicity, but if it approximates the requirements mentioned and if it yields good results, some extra effort on the part of the student to master it is well applied. On the other hand, a method which has been simplified out of all accuracy is worse than useless. It cannot be presented to the student without apology; but an apology for a poor

[1] One of the best treatments of the question which the writer has seen appears in the Zeleny and Erikson manual, 5th ed., p. 67. In a private communication Professor Erikson states that the method there described is due to Prof. L. F. Miller. The method here presented resembles that of Miller in some respects.

experiment cannot take the place of results which establish in the student's mind the assurance that proper evaluation of significant experimental data usually yields accurate results.

According to Ostwald-Luther, accuracy better than 1% in calorimetric experiments requires painstaking precautions both in the experimental work and in the treatment of the observed values. Assuming that the data have been obtained by the student with due care, his results will almost certainly be several per cent in error unless he has a suitable method for making his correction to the observed temperature rise. This correction is therefore comparable in importance with the primary object of the experiment. An inadequate or too greatly simplified method, which misses the correction by only one-tenth of a degree may leave an error in the final result much greater than the one per cent mentioned.

The method here presented, in common with most methods, is derived from Newton's law of cooling. Its principal novelty is the manner in which application of the law is made. Briefly, it consists of (1) finding the rate of temperature change of the calorimeter before, during and after the intended heat input by making temperature measurements at reasonably short intervals; (2) using the temperature data in a graphical method for determining the rates of temperature change in the initial and final periods of the experiment; (3) from an auxiliary construction, finding the average rate of temperature change during one or several periods of time during the intended heat input, and the corresponding time intervals; (4) finding the correction to the observed temperature rise by algebraic summation of the product of each rate of temperature change and its corresponding time interval.

Description of Method: A simple case has been chosen to illustrate the procedure in finding the temperature correction. The curve of temperature change with time for this case is shown in Fig. 1. The usual three periods of the experiment are represented by *LM*, *MN*, and *NO*, respectively. Observations for the curve may be taken at suitable intervals, 10 or 15 seconds being convenient if one student makes the observations while the other calls for and records readings. The rate of heat input during the period represented by *MN* was approximately constant, so that practically *MN* is a straight line. Obviously, if *MN* were not a straight line it could be divided into sections, each of which could be represented by a straight line approximation.

Although the theory of the method does not require that the temperature rise of the calorimeter be such that the temperature of the surroundings is approximately in the middle of this interval, it is desirable that this condition be approximated more or less closely, because the corrections thereby become smaller and, in part, compensating. It is important to note, however, that the temperature of the surrounding air does not enter into the data from which the corrections are found.

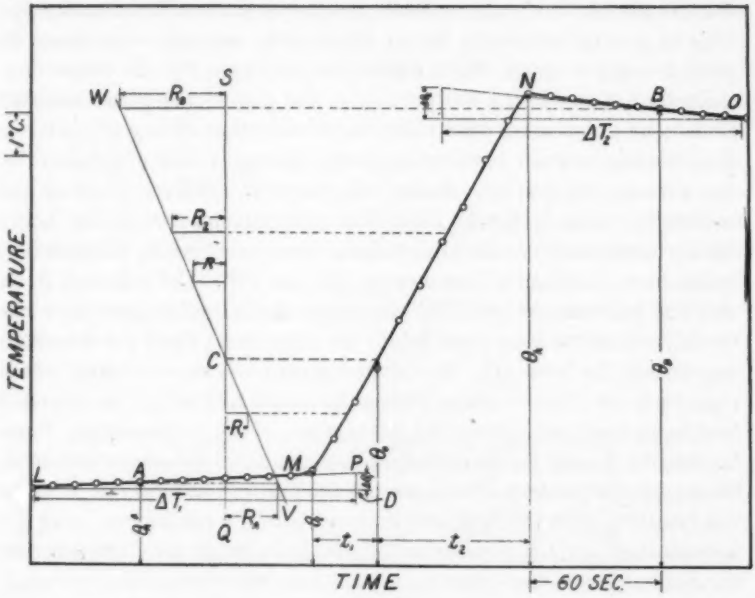

FIG. 1.

At any suitable point A, corresponding to temperature θ_A, determine the initial rate of temperature change by drawing the tangent LP through the point A. Then obtain the numerical value of this rate by finding the ratio $\Delta\theta_1/\Delta T_1$. Let this value be designated R_A. At the point B, corresponding to the temperature θ_B, find R_B, the ratio of $\Delta\theta_2/\Delta T_2$ in the same manner.

In the space to the left of the graph make an auxiliary construction as follows: Draw the vertical axis QS, having the same temperature scale as the graph, and project to this axis the temperatures θ_A and θ_B by drawing horizontal lines as shown. Lay off R_A on any convenient

scale on a horizontal line having the ordinate of its corresponding temperature θ_A, and lay off R_B in the same manner. These distances will lie on opposite sides of the axis QS if the initial change was a temperature rise and the final change a temperature fall. In case the temperature changes are in the same direction R_A and R_B should, of course, be laid off on the same side of the axis QS.

Through the extremities of R_A and R_B draw VW. The point C where it intersects the axis QS then represents the temperature θ_0 at which there is not heat exchange between the calorimeter and its surroundings. This in general will differ by an appreciable amount—sometimes by several degrees—from the temperature indicated by the room thermometer.[2] Considering the portion of the graph during the intended heat input which takes place below θ_0, we note that during this interval the calorimeter must have been gaining heat at a rate determined by the average temperature during this interval. This average is the arithmetic mean of θ_i, the beginning temperature, and θ_0, the equilibrium temperature. At the ordinate corresponding to this average value draw a horizontal line through QS and VW. The intercept R_1 on this line between QS and VW represents the rate of temperature rise resulting from the heat passing into the calorimeter from the surroundings during the interval t_1. In a similar manner R_2 may be found, which represents the rate of cooling during the period t_2 in which the intended heat input took place above the temperature of the surroundings. From the lengths R_1 and R_2 the corresponding numerical values are obtained. Obviously the product t_1R_1 represents the part of the total temperature rise resulting from the heat which passed into the calorimeter from the surroundings and t_2R_2 represents the deficiency in the total temperature rise resulting from the loss of heat from the calorimeter to the surroundings. Hence t_1R_1 must be subtracted from the total temperature rise $\theta_h-\theta_i$, and t_2R_2 must be added to this difference.

In the case illustrated where MN is appreciably a straight line it would not have been necessary to construct R_1 and R_2 separately; R, which is the mean value of R_1 and R_2, might have been determined directly by finding the intercept between VW and QS at the average temperature $(\theta_h+\theta_i)/2$. The total time interval t, which is the sum of t_1 and t_2, multiplied by R would, of course, give the same result. In the illustration the process of finding the temperature correction was

[2] Such difference is, of course, to be expected. The "temperature of surroundings" will depend on conditions other than merely the temperature of the surrounding air.

presented, for the sake of greater clearness, as consisting of a determination of the two separate values.

When the rate of rapid temperature rise is not uniform, the curve of temperature rise is approximated by a broken line. In this case, the total correction $\sum tR$ is the algebraic sum of as many corrections of the form tR, as there are straight-line segments. The sign of each correction is determined according as the corresponding R lies to the right or left of the vertical line QS. The individual correction is additive when R lies to the left, and subtractive when R lies to the right of QS.

The process described in connection with Fig. 1 may be summarized in the equation

$$\theta = (\theta_h + t_2 R_2) - (\theta_l + t_1 R_1).$$

Table 1 illustrates the efficacy of the present method in correcting for the heat exchange between the calorimeter and its surroundings in the well known method due to Puluj for determining the mechanical equivalent of heat. The experiment was repeated eight times and the

TABLE 1

I	II	III	IV	V	VI	VII	VIII	IX
1	21.0	7.56	430	7.24	449	7.38	425	−0.5%
2	22.8	8.22	432	8.26	434	8.25	431	+0.9%
3	21.5	9.00	423	8.76	434	8.84	430	+0.7%
4	21.5	7.68	416	7.25	441	7.43	430	+0.7%
5	21.0	8.28	414	8.06	425	8.18	421	−1.4%
6	20.2	9.10	410	8.54	437	8.83	420	−1.6%
7	22.2	11.50	414	10.97	434	11.23	425	−0.5%
8	20.9	3.84	418	3.71	432	3.75	427	0.0%
Averages:....			419.6	....	435.7	....	426.1	−0.2%

KEY

Column I: Number of experiment.
Column II: Temperature of room.
Column III: Temperature rise corrected by Ferry's method.
Column IV: Mechanical equivalent of heat from temperature in Col. III.
Column V: Temperature rise, uncorrected.
Column VI: Corresponding J in g-m/cal.
Column VII: Temperature rise corrected by method described in present article.
Column VIII: Corresponding J in g-m/cal.
Column IX: Per cent error in final result for J.

temperature correction for heat exchange was first found by Ferry's modification of Rowland's method. Column II is the reading of a thermometer near the apparatus and these values were employed in determining the correction by Ferry's method. The corresponding corrected values for the temperature rise are given in column III. Column IV gives the corresponding values of the mechanical equivalent of heat when the values for temperature rise shown in column III were used. The disagreement between these values and the accepted value of 427. g-m/cal, was so pronounced that it initiated the study which resulted in the method here described.

The first step in the study was to evaluate the data without correcting for the heat exchange. The uncorrected temperature rises in each of the eight experiments are shown in column V, and the corresponding values of J in VI. The average of column VI, 435.7, is almost as near 427 as that of column IV, but departure is in the opposite direction. Evidently Ferry's method resulted in overcorrection for heat exchanges.[3]

Column VII shows the values of temperature rise determined by the method here described. These may be compared with column III and V. The corresponding values of J are given in column VIII, the average of which is 426.1; and column IX shows the percentage error in each determination. The table indicates that with the same data, the present method yields results more accurate than those obtained by the other methods used, and that the value obtained in any one of the eight determinations would have been acceptable in a student's laboratory report.

SUMMARY

A description is given of a method by which the temperature rise in calorimetric experiments can be found, yielding results which are as accurate as ordinary calorimetric data can be expected to give. This is accomplished by determining graphically the rate of temperature change in the calorimeter resulting from heat exchanges with the surroundings. The method is an application of well known principles in calorimetry. In a test of the method results are shown for a series

[3] It should be said that in applying Ferry's method it was found difficult to control conditions so that room temperature was in the middle of the interval of rapid temperature rise. The graphical construction here described shows, in all eight experiments, that the "temperature of surroundings" as deduced from the data differs from "room temperature" by appreciable amounts. Since the "temperature of surroundings" cannot be predetermined, and since this temperature ought to be that below and above which, in Ferry's method, the temperature changes of the calorimeter should be equal, it is difficult to obtain data—at least in mechanical equivalent of heat experiments—to which Ferry's correction is applicable.

of determinations of the mechanical equivalent of heat for which the average value is 426.1 g-m/cal as against 419.6 when another method of correction was used with the same data, and 435.7 when the temperature rise was not corrected.

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International Critical Tables of Numerical Data of Physics, Chemistry and Technology. Prepared under the auspices of the International Research Council and the National Academy of Sciences by the National Research Council, Edward W. Washburn, Editor in Chief. Published for the National Research Council by the McGraw-Hill Book Co., New York. Sold only in sets, five volumes, $60.00—$12.00 per volume as issued. Vol. I : xx + 415 pages.

The appearance of this, the first volume of the International Critical Tables has been awaited with much interest, since it gives to the scientific public the first real glimpse of this very ambitious undertaking. Categorically, this first glimpse is very satisfying, for it shows how thoroughly the Editorial Board has accomplished its task, and how valuable the entire work bids fair to be in furthering scientific and technical work of all kinds.

A large part of this first volume—some 250 pages—deals with the Physical Properties of Chemical Substances. The material in this section is grouped into three tables, with numerous sub-divisions, wherein one can find the physical constants of thousands of organic and inorganic substances—and of course of the elements. The data is so arranged that any compound can be found either by name, or by chemical formula. Further, in supplementary tables with adequate cross-references, substances are arranged in the order of magnitude of a particular property, as refractive index, melting point, boiling point, etc. Thus some 800 organic liquids are listed in the numerical order of their index for the D line (dispersion also given) for values of the index running from 1.306 to 1.6913. Or, one finds that there are 31 different substances listed, whose melting points are between 56° C and 57° C. These tables are valuable for identification of unknowns or for choosing substances with particular properties. A complete description of this section of Vol. I is impossible in a brief review.

Among other sections of this volume may be mentioned Laboratory Technique (thermometry, production of constant temperatures, and constant humidity, calibration of volumetric vessels, high vacuum technique, etc.); Radioactivity; Crystal Structure; Astronomical and Geodetic Data; Aerodynamics; National and Local Weights and

Measures; Symbols, Basic Constants, Conversion Data, Definitions; and The Chemical Elements and Atoms.

The data given are "critical" which "means that the Cooperating Expert was requested to give the *best* value which he could derive from all the information available · · · ·" Some fifty Cooperating Experts from seven different countries collaborated with the editorial board in preparing this volume, and these were assisted by numerous other workers. Many parts of the text are given in English, French, German and Italian in parallel columns.

The compilation of the data for the five volumes was made possible by subscriptions totaling $170,000 from some 244 firms and individuals.

Criticism of such a work as this cannot be based on casual inspection. Shortcomings, if any there be, can be brought to light only by actual use. The only criticism suggested by this short review of the volume is that the meaning of the symbols used in some of the tables is not *easily* discovered from the text, a circumstance which may to a slight extent impair the value of the book as a *ready* reference.

The Editors, their many collaborators, and the Publishers are to be congratulated on bringing to completion a work of such far-reaching proportions, the value of which to Science and to Industry can hardly be over estimated.

F. K. RICHTMYER.

Control of Optical Surfaces.—This fourth installment in the series on testing methods deals with the celebrated knife-edge shadow method of Foucault, and with the variations that have recently been made in it by Ronchi and by Lenouvel. If a movable slit were substituted for the Foucault knife-edge the surface under examination would appear bright wherever the value of the tangential error corresponded with the measurable lateral decentration of the slit. And if the slit were replaced by a series of fixed slits, that is, by a suitable line screen, the surface would present a striated appearance, the location of the light and dark bands being determined by the errors of the wave and by the position and spacing of the screen. The geometrical development of the subject is given in detail. To complete the study it is necessary to consider the physical optics of the problem. A fixed wedge producing interference fringes may be used instead of the screen and has the advantage that it fixes the values of the tangential errors used. Unfortunately, it restricts the control of marginal zones when the sensitivity of the method is increased. The method gives promise in the commercial testing of photographic objectives, but does not appear to be adapted to the precise control of astronomical systems. [G. Yvon, Revue d'Optique, *4*, pp. 345–377; 1925.]

G. W. MOFFITT

A METHOD OF OBTAINING LIGHT FLASHES OF UNIFORM INTENSITY AND SHORT DURATION

By J. W. Beams*

In the study of the successive appearance of spectrum lines in spark and condensed discharges[1,2], it is desirable to study not only the beginning of a discharge but also any part of a discharge without the masking effect of the preceding and following stages. Also in determining the time between excitation and emission in fluorescence[3] as well as the time that the emission continues after the exciting light has been cut off, it is necessary to have an optical shutter which will open abruptly, remain open any desired time from 10^{-9} to 10^{-7} seconds and then close abruptly. For these purposes the method herein described has been developed.

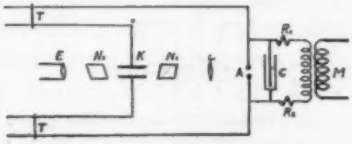

Fig. 1. *Electro-optical shutter.*

An electro-optical shutter (Fig. 1) which closes very abruptly has been previously described[1]. The arrangement is illustrated in Fig. 1. K is a "Kerr Cell" made by immersing two parallel plates of metal in a tube of carbon bisulphide. N_1 and N_2 are crossed Nicols, with their diagonals at 45° with the plates of K. If an electric field is applied across the plates of K, the carbon bisulphide becomes doubly refracting[4] and light passes N_2. If however the field across K is relaxed no light passes N_2. It was found that if the field in K was relaxed by the spark at A no light passed N_2 provided the leads from A to K were very short. By lengthening the lead wires from A to K light from the spark at A passes N_2. N_1KN_2 then is in effect, an electro-optical shutter, the closing of which can be advanced or retarded with respect to a spark by

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[1] Brown and Beams, J.O.S.A. &R.S.I., *11*, p. 11; 1925.

[2] Beams, Phys. Rev. *28*, 8475; 1926

[3] Hoxton and Beams, Abst. Phys. Rev., *27*, p. 245; 1926.

[4] Kerr, Phil. Mag., *1*, pp. 337, 446; 1875; *7*, pp. 85, 229; 1879.

lengthening or shortening symmetrically the lead wires from A to K. In this shutter it will be noted that the field across K is applied slowly and hence the shutter opens slowly, its closing only being abrupt.

In the present arrangement (Fig. 2) S is a steady light source. N_1 and N_2 are crossed Nicol prisms, the diagonals of N_1 making 45° with the vertical. K_1 and K_2 are Kerr cells of practically identical dimensions (in most of these experiments each plate was 12 cm by 1 cm and the plates were 0.45 cm apart). The plane of the plates of K_1 was vertical while that of K_2 was horizontal. A is a spark gap which releases the electric field across K_1 and K_2. Connected across A is a source of high potential such as a 10000 volt 60 cycle transformer or induction coil. T_1T_1 and T_2T_2 are cross wires by which the lead wires from A to K_1 and from A to K_2 can be lengthened or shortened by the observer at E. If an electric field is slowly applied across A the electric fields in K_1 and K_2 are equal during the time of charging and hence,

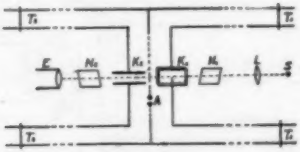

FIG. 2. *Electro-optical shutter which opens and closes abruptly.*

since K_1 and K_2 are identical and at right angles, the electric double refraction in K_1 is exactly compensated by that of K_2 and no light from S passes N_2. In the case of a sudden discharge of K_1 and K_2 through the spark gap A no light will pass N_2 if the lead wires from A to K_1 are equal and symmetrical with those from A to K_2, *i.e.*, the electric double refraction in K_1 at every instant is compensated by that in K_2. On the other hand if, by lengthening the lead wires from A to K_2, K_2 is made to discharge a certain time after K_1, light will pass N_2, *i.e.*, $N_1K_1K_2N_2$ acts as a light shutter which opens abruptly, remains open any desired time from 10^{-9} to 10^{-7} seconds and then closes abruptly. It will be noted that the charging of K_1 and K_2 is very slow in comparison to the discharge. This is necessary in order that K_1 and K_2 will be at the same potential during the charging when their lead wires to A are unequal. On the other hand it is desirable to charge them as many times per second as possible because this increases the number of light flashes and hence lessens errors of observation. The time required

for the opening and for the closing of the shutter of course depends upon the capacity of K_1 and of K_2 and upon the length of the lead wires. With the dimensions used, a rough photometric study of the rate of increase of intensity of the light during the opening of the shutter shows that it completely opens in 10^{-9} seconds, and over ranges so far investigated the closing is almost as abrupt. The very quick opening and closing of this shutter results from the fact that the electric double refraction varies as the square of the electric field[4], and that the damping is sufficient to stop effective oscillations in K_1 and K_2.

The intensity of the light flashes depends upon the strength of the electric field in K_1 and K_2 and upon the intensity of the source S. A "tungs-arc" was used at S in most of the experiments, but it is not intense enough for many purposes.

The most intense source so far tested was made by picking out certain portions of the electric spark A placed at S. The brightness of A can be increased by placing Leyden jars in parallel with it and choosing desirable metals for electrodes. For light flashes over 5×10^{-8} seconds in length, unless considerable capacity is placed in parallel with A, the spark is unsatisfactory because of the variability of the intensity of the spark discharge. If a spectroscope is placed at E and the spectrum of the spark, say of zinc, examined, it is found that the air lines appear first with high intensity but very soon fade out. The spark lines next appear, followed by the arc lines, the spark lines fading out after the arc lines begin. It was found possible to pick out the (4912,24) spark lines of zinc practically alone and at this stage the zinc spark is probably at maximum brightness. The above observations are in general accord with those made by other methods.[5] A closer examination of both spark and condensed discharges is soon to be made.

Since the carbon bisulphide is opaque to the ultraviolet, a study of liquids to take its place has been made. It was found that chloroform made an excellent substitute for the carbon bisulphide. When pure it is a good insulator and has a negative Kerr constant[6] almost equal in absolute value to the positive Kerr constant of carbon bisulphide. Photographs of lines as short as 2558 AU have been made through the shutter when the Kerr cell was filled with chloroform and the Nicol prisms replaced by Foucault prisms. As is well known, if large Focault prisms are not available the Canada balsam in the Nicol prisms can be replaced by glycerine for work in the ultraviolet.

[5] See Baly, "Spectroscopy," p. 406, Longmans, 1912.
[6] Schmidt, Ann. d. Physik, 7, p. 142; 1902.

The question naturally suggests itself as to how long the electric double refraction in K_1 and K_2 remains after the electric field is removed. The above arrangement gives a method of determining the difference in this lag of the Kerr effect in any two liquids. A very rough test was hurriedly made with carbon bisulphide and chloroform. The zinc spark lines (4912,24) were used as a light source. K_1 and K_2 were first filled with carbon bisulphide and the lead wires so adjusted that no light passed N_2. The carbon bisulphide was then removed from K_2, chloroform was substituted, and the plane of the plates of K_2, rotated approximately 90° until complete compensation was secured while the cells were being charged. The arrangement, thus modified permitted light to pass N_2 but that a slight shortening of a few centimeters of the lead wires AT_2K_2 lessened its intensity, a minimum occurring at about one hundred centimeters decrease of lead wire. The reason that the light could not be completely extinguished when chloroform was substituted for carbon bisulphide was that the time of discharge of K_2 was changed (because of the difference in dielectric constants of carbon bisulphide and chloroform). This source of error however is very small and could not completely account for the observed lag. However the experiment definitely shows that the difference in the lag of the Kerr effect in carbon bisulphide and chloroform is at most very small, and less than $4 \cdot 10^{-9}$ seconds if we assume that the fall of potential travels along the lead wires at about the velocity of light. The writer hopes to carry out further experiments with different liquids under better experimental conditions.

In conclusion, it is a pleasure to acknowledge my indebtedness to Professor L. G. Hoxton for many valuable suggestions and criticisms, also to Messrs. A. T. Bishop and T. E. Gilmer for aid in investigating liquids for use in the ultraviolet, and to Mr. A. J. Weed, instrument maker, for help in the construction of apparatus.

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CHARLOTTESVILLE, VIRGINIA, JULY 8, 1926.

AN ACCESSORY DEVICE FOR POTENTIOMETERS

By Perry A. Borden

Apropos of Messrs. Worthing and Forsythe's description of modifications in the Leeds and Northrup Type "K" potentiometer, appearing in the May 1925 issue of this Journal, attention may be called to a very similar modification developed by the writer in 1922, and since found very useful, particularly in checking the accuracy of wattmeters, but also in such other work as precise measurement of low resistance values.

This method differs materially from the system cited, in only one particular; and that is that there are required absolutely no structural changes in the potentiometer, all auxiliary circuits and contacts being carried on a separate unit, which consists essentially of a double-

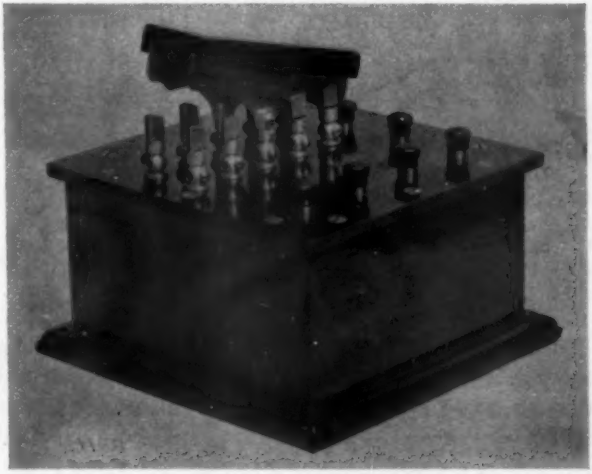

Fig. 1. *Auxiliary switch for use with Leeds and Northrup potentiometer.*

throw four-pole switch, as illustrated in Fig. 1. The circuits are as shown in Fig. 2, wherein it will be seen that all connections are made to regular terminals of the instrument, with the addition that the short flexible lead supplied by the makers for calibrating is made use of, in connecting to the proper point on the main dial of the slide-wire. In the potentiometer, as regularly built, the tap for the standard cell is brought from the point ".5" on the slide-wire, and, when the throw-over switch is in the proper position, passes through the galvanometer circuit and keys. If, then a tap be taken from the main dial at any

point above ".5", the potential from it, through the galvanometer circuit will be $E-.5$, where E is the tap chosen. By means of the auxiliary switch shown, this change may be quickly made, and the standard cell replaced by an electromotive force the value of which depends upon the tap chosen.

As an example of the use of this system, the following is given as the procedure in checking an indicating wattmeter: The voltage circuit of the instrument is maintained at a potential of 100 volts; and means for regulating this is placed within reach of the potentiometer operator. Current from a separate source is supplied to the current coils of the wattmeter, and placed under control of the meter reader. The movable plug is placed in the "1.5" point on the dial. The voltage across the wattmeter terminals, reduced by a 100/1 volt-box, is led

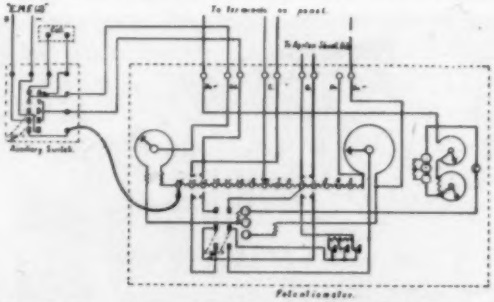

FIG. 2. *Use of auxiliary switch with Leeds and Northrup potentiometer.*

to the "emf (2)" terminals of the auxiliary switch, while the current, passed through a suitable standard resistance, is tapped off to the emf terminals of the potentiometer. Both switches being set to the standard cell, the slide-wire current is then adjusted in the ordinary way, after which the auxiliary switch is thrown to emf (2), which brings in the wattmeter voltage against a fixed potential of 1 volt on the slide-wire. The rheostat controlling this voltage is then adjusted until its value is correct, the instrument reader meanwhile holding himself in readiness to make the final setting on the wattmeter. This may be usually accomplished in a very few seconds after the voltage is correct. The potentiometer switch is then thrown over to emf, and the value of the wattmeter current read on the slide-wire. If any doubt exists as to the voltage having remained steady, the readings may be repeated without disturbing any of the settings.

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APPARATUS FOR DETERMINING OXIDATION-REDUCTION POTENTIALS

By John M. Ort

Clark and his associates[1,2,3,4,5,6,7,8] have pointed out some of the possibilities of the application of measurements of oxidation-reduction potentials to the solution of certain biologic problems. They have discussed in some detail the theoretic points involved, have described an apparatus for this work, and have given data for several classes of compounds. Their papers also contain a rather extensive bibliography for this field of research. Kendall and Ort[9] have applied Clark's methods to the investigation of certain compounds closely related to thyroxin, and the data so obtained have suggested a new theory of the action of thyroxin in the body. Work in progress indicates that this method of attack will continue to prove fruitful in other biologic problems.

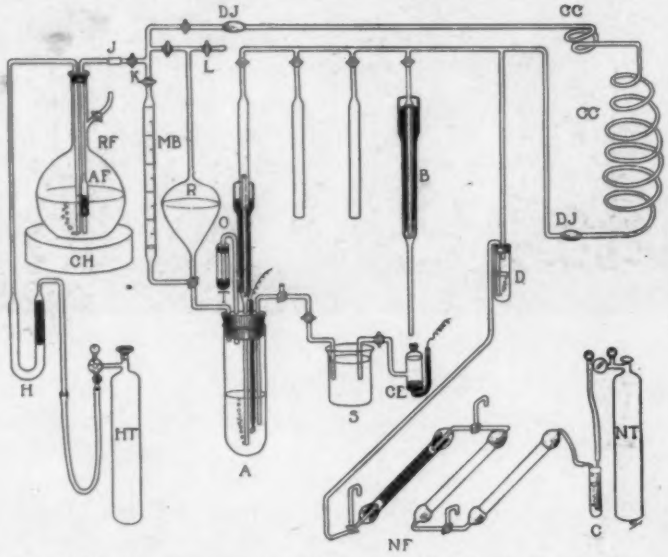

FIG. 1. *Diagrammatic arrangement of apparatus for the determination of oxidation-reduction potentials.*

The essential features of the apparatus used for this work are represented in diagrammatic form in Fig. 1, and the entire arrangement is shown photographed in Fig. 2. The solutions to be investigated are placed in four electrode chambers. Some of the details of these cham-

bers with their electrodes and connections are shown photographed in Fig. 3. The long mercury seals above the chambers permit them to be raised for observation or for cleaning and changing solutions, and to be lowered into the oil bath for reading. In any position there is an all-glass connection to the nitrogen line. The details of one of these chambers in the lowered position are seen at *A* (Fig. 1). *B* shows the relation of the mercury and tubes when the seal is raised.

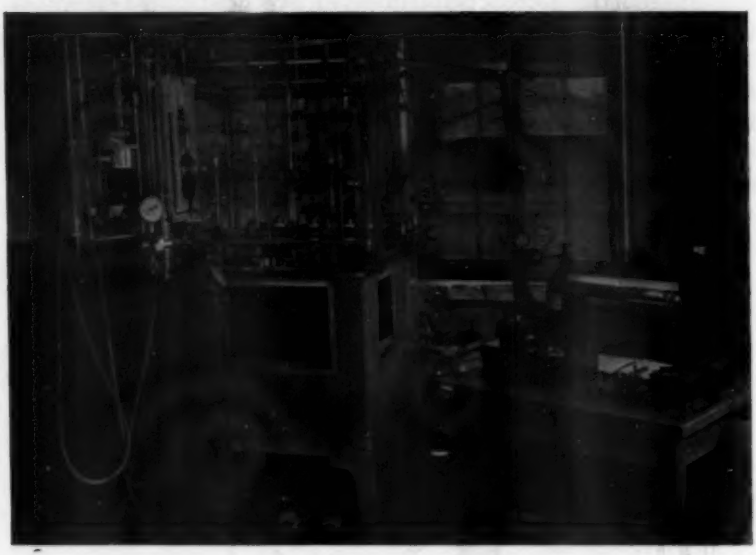

FIG. 2. *Complete apparatus.*

By means of the reduction flask *RF* and the movable burette *MB*, solutions can be reduced, stored, freed from all gases except nitrogen, and transferred to the electrode chambers out of contact with air. With the tip *T* as shown, the movable burette is in a position to deliver a solution into *A*. By means of the copper coils *CC* and *cc*, a flexible gas-tight connection is provided which allows the moving of the burette, which is supported from above by an iron pipe, *T*, that slides along a horizontal pipe (not shown) and so permits delivery of solutions to any one of the four chambers. The joints connecting the copper tubes to the glass tubes are covered with De Kotinsky's cement and are indicated at *DJ*.

With the exception of the copper coils, all of the apparatus thus far mentioned is shown in Fig. 2, mounted on top of a large Freas water

thermostat. To prevent leakage of electricity, a high flash-point signal oil is used instead of water. The use of oil made it necessary to remove the putty from the glass windows and to reset them in a cement of litharge and glycerin. This thermostat, with its oil bath, maintains the temperature constant to within $\pm 0.01°C$.

The necessary removal of all gases, such as oxygen and hydrogen, that are not inert is accomplished by flushing them out with pure nitrogen. Commercial nitrogen in a tank *NT* (Fig. 1) is first passed through strong sodium hydroxid in *C* to remove any carbon dioxid and then through the three electric furnaces *NF* to remove all traces of oxygen. The furnaces contain copper reduced from copper oxid wire. They are composed of glazed silica tubes wound with nichrome wire and covered with asbestos. The connections between these tubes are of pyrex glass tubing in which are sealed three-way glass stopcocks. The glass tubes are mechanically supported in the ends of the silica tubes by tightly-fitting rubber stoppers pushed about one inch into the tubes. The ends beyond the stoppers are covered with DeKotinsky's cement to prevent the possibility of oxygen from the air diffusing through the stoppers. These furnaces remove all oxygen from the commercial nitrogen used, even when the gas passes through at the rate of more than a liter a minute. Once or twice a week the furnaces are reduced by passing hydrogen through them. The extent of oxidation of the copper can be determined by catching the water coming from each furnace at the three-way stopcocks during the reduction. From furnaces the nitrogen goes through very dilute sodium hydroxid at *D*, to nearly saturate it with water vapor, and then to the various parts of the system as shown. In Fig. 2 the furnaces are shown on the floor beneath the thermostat.

It has sometimes been necessary to add solutions of reduced dyes to the electrode chambers during a run. The reduction of the dyes is carried out in the 500 cc pyrex flask, *RF* (Fig. 1). It has a stopcock attached to its neck and can be heated by a Cenco electric heater, *CH*. The hydrogen for the reduction is led from the tank below, *HT*, through a small electric furnace, *H*, also wound with nichrome wire and filled with fine pieces of metallic copper reduced from copper oxid wire. The hydrogen, freed from oxygen, passes through the tube in the rubber stopper in the neck to the bottom of the reducing flask. After bubbling through the dye solution and suspended finely-divided platinum, the hydrogen passes out of the flask through the asbestos and glass-wool filter *AF*, in the other tube. (This escape of hydrogen could not take place with the flask and *AF* in the position for delivery

as shown. The drawing shows the flask, *RF*, lowered to deliver solution to *MB*. This is described later.) Any solution can be added to the reduction flask through the stopcock on the side tube.

The hydrogen from the reducing flask passes to the movable burette, and its 500 cc reservoir, *R*. After the reduction is complete, and the

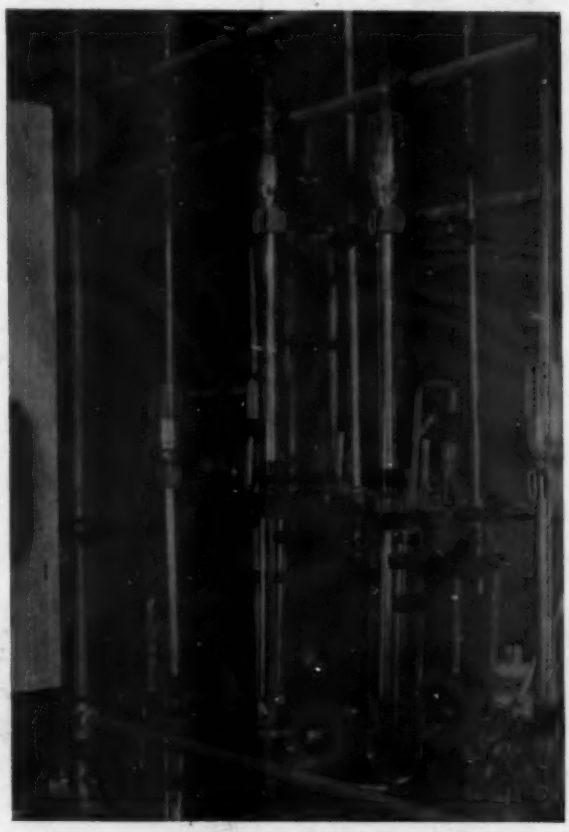

FIG. 3. *Details of electrode chambers and mercury seals.*

hydrogen has driven all air out of the movable burette and reservoir, the reduced solution is forced from the reduction flask through the burette into the reservoir. This is accomplished as follows: The reduction flask with its supporting pipe frame is moved up, while the tube with the asbestos filter is held stationary, the rubber stopper in the reduction flask being slipped along the tube. The end of the filter

tube, which before has been above the surface of the liquid in the flask to allow the escape of the hydrogen, is thus put below the surface, and so the hydrogen pressure forces the solution of the reduced dye along the path taken before by the hydrogen. (The apparatus is shown in this position in both Figs. 1 and 2, just after the delivery of a solution to the movable burette and reservoir.) The burette is then shut off from the air and reduction apparatus by closing the stopcocks *K* and *L* (Fig. 1), at the top, and the whole reduction apparatus removed as a unit, coming apart at the joint, *J*.

Since the oxidation-reduction potentials of solutions depend to some extent on the hydrion concentration, all determinations are made in heavily buffered solutions. Sometimes these can be added directly to the electrode chambers along with the compounds investigated. At other times these buffers must be first put into the movable burette for removal of dissolved air. In such cases the buffers are sucked into the burette through the tip, *T*, deoxygenated, and then delivered to the electrode chambers.

The electrode chambers, *A* (Fig. 1), are simply large pyrex test-tubes about 9 inches long, fitting the No. 10 rubber stoppers at the top. A gas outlet is provided at *O* through a mercury trap. When the tip, *T*, of the movable burette is withdrawn from the stopper, the hole is closed by a glass plug. Extending down into the chambers nearly to the bottom are the platinum electrodes, the glass tube of the nitrogen inlet, and the glass tube leading to the salt bridge. In this latter tube, outside the chamber, are one three-way stopcock and one straight stopcock. When the chamber is lowered into the oil, the end of this tube dips into a beaker, *S*, of saturated potassium chlorid supported in the oil. Into this beaker three saturated calomel electrodes also dip. (Only one, *CE*, is indicated.) Connection from the electrode chamber to the beaker is made by applying suction on the side tube of the three-way stopcock and drawing up the potassium chlorid solution through both stopcocks and then closing the straight one. By turning the three-way cock some of the solution is next drawn up from the electrode chamber and then this stopcock is also closed, making the union in the three-way cock. In this way contamination of both the solutions in the chamber and in the calomel electrodes is prevented. The three calomel electrodes have stood for nearly a year without varying more than a few tenths of a millivolt from each other or from the correct value as checked by hydrogen electrode determinations.

The potentials were read with a Leeds and Northrup type *K* potentiometer which is shown on the table in Fig. 2. The telescope and scale

are shown above it. A Leeds and Northrup high sensitivity galvanometer (not shown) is used, although so sensitive an instrument would not be necessary for this particular kind of work, since results are reported only to the nearest millivolt. Two Weston standard cells, calibrated by the United States Bureau of Standards, are the voltage standards.

With an arrangement such as the one described, measurements of oxidation-reduction potentials can be very conveniently carried out. Oxidizing, reducing, or neutral solutions can be prepared, stored, and transferred to the electrode chambers at any time without any contact with air during the run. Compounds, that when moist, are affected by the oxygen of the air, can be put into the chambers in the dry form, the air removed, and then deoxygenated solutions added and the measurements taken. This was done throughout the work with thyroxin derivatives. And, at any time during a run, the electrode chambers can be lifted out of the bath for close observation, such as to determine dye colors, and then be put back without any disturbing results.

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BIBLIOGRAPHY

1. Clark, W. M., Studies on oxidation-reduction. I. Introduction. Public Health Reports, *38*, pp. 443–55; 1923.
2. Clark, W. M. and Cohen, Barnett, Studies on oxidation-reduction. II. An analysis of the theoretical relations between reduction potentials and pH. Public Health Reports, *38*, pp. 666–83; 1923.
3. Clark, W. M. and Cohen, Barnett, Studies on oxidation-reduction. III. Electrode potentials of mixtures of 1-naphthol-2-sulphonic acid indophenol and the reduction product. Public Health Reports, *38*, pp. 933–58; 1923.
4. Sullivan, M. X., Cohen, Barnett and Clark, W. M., Studies on oxidation-reduction. IV. Electrode potentials of indigo sulphonates, each in equilibrium with its reduction product. Public Health Reports, *38*, pp. 1669–1718; 1923.
5. Cohen, Barnett, Gibbs, H. D., and Clark, W. M., Studies on oxidation-reduction. V. Electrode potentials of simple indophenols, each in equilibrium with its reduction product. Public Health Reports, *39*, pp. 381–414; 1924.
6. Cohen, Barnett, Gibbs, H. D., and Clark, W. M., Studies on oxidation-reduction. VI. A preliminary study of indophenols: (a) dibromo substitution products of phenol indophenol; (b) substituted indophenols of the ortho type; (c) miscellaneous. Public Health Reports, *39*, pp. 804–23; 1924.
7. Gibbs, H. D., Cohen, Barnett and Cannan, R. K., Studies on oxidation-reduction. VII. A study of dichloro substitution products of phenol indophenols. Public Health Reports, *40*, pp. 649–63; 1925.
8. Clark, W. M., Cohen, Barnett and Gibbs, H. D., Studies on oxidation-reduction. VIII. Methylene blue. Public Health Reports, *40*, pp. 1131–1201; 1925.
9. Kendall, E. C. and Ort, J. M., The oxidation-reduction potentials of 2 oxo dihydro-indol 3 propionic acid and some of its halogen derivatives. Jour. Biol. Chem., *68*, pp. 611-630; 1926.

MEASUREMENT OF REVERBERATION USING THE THERMIONIC TUBE OSCILLATOR AS A SOURCE*

By Vern O. Knudsen

If W. C. Sabine had begun his investigation of the acoustics of interiors today instead of nearly thirty years ago, he probably would have utilized thermionic tube circuits in his investigations. Thirty years ago a set of organ pipes was the most satisfactory available apparatus for producing nearly pure tones of the pitch and loudness range required for the investigation of acoustic problems. Naturally, therefore, organ pipes were used by Sabine, and they have been used by his successors.

The development of the thermionic tube and thermionic tube circuits during the past ten years has made available more convenient and precise apparatus for acoustic meaurements. For example, the vacuum tube oscillator, with appropriate filters and a high quality receiver, will produce approximately pure tones of any required pitch and loudness, and the pitch and loudness can be readily and accurately determined.

The method developed by W. C. Sabine for determining the initial intensity of sound in a room just before an organ pipe is stopped, is as follows.[1] One, two, three and four pipes, all of the same pitch and all connected to the same pressure tank, are successively sounded in all possible combinations, and the corresponding times of reverberation measured. From these data the corresponding decay curves are plotted, and the initial intensity of the sound from one, two, three or four pipes is obtained by extrapolating these curves to zero on the time axis.

Anyone who has made reverberation measurements will appreciate the extreme care and the patient labor which was exhibited by Sabine in conducting these reverberation experiments. Considering the small range of intensity variation available with only four pipes, his precise results merit great respect.

Anyone who has made reverberation measurements with organ pipes will also appreciate the limitations and disadvantages inherent in the apparatus and method. In the first place, the calibration of the intensity of the test tones is cumbersome and slow, and requires

* Presented to the Stanford meeting of Amer. Phys. Soc., Mar. 6, 1926.

[1] W. C. Sabine, Collected Papers on Acoustics, Harvard Univ. Press.

great care and accuracy. Secondly, since the apparatus is calibrated under very quiet conditions, any residual noise in a less favorably situated room produces a masking effect upon the tone which introduces an error in determining just when the test tone has been reduced to minimal audibility. When measurements are made in other rooms than the test room, some residual noises probably are present, and to minimize the errors from this source, intensity calibrations, based upon minimal audibility under the conditions of the tests, should be made during the progress of the reverberation measurements. The time required for intensity calibrations of a set of organ pipes makes this procedure prohibitive. Thirdly, the harmonics in the organ pipe notes, under certain conditions, may be undesirable. Lastly, the apparatus is not very portable.

For the past three years the writer has been using a thermionic tube oscillator as a source of tone for reverberation and sound absorption measurements. This apparatus overcomes most of the limitations and disadvantages of the organ pipes, and it seems advisable therefore to present a brief description of the apparatus and method. The general arrangement of the apparatus is shown in Fig. 1. The oscillator will produce frequencies throughout the entire audible range. The in-

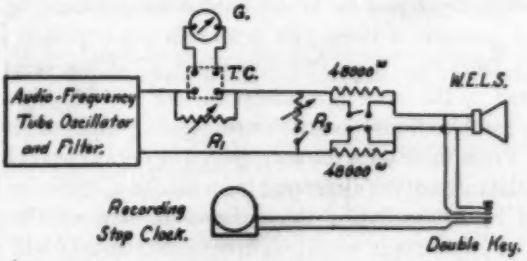

FIG. 1. *Thermionic tube oscillator and auxiliary equipment for reverberation measurements.*

ductances are of the air core type, the condensers are either of the mica or air type, and the grids of the tubes are maintained at zero or negative d. c. potentials. The impedances in the tuned oscillating circuit are therefore nearly independent of the currents flowing through them, and hence the frequencies of the various tones produced by the oscillator remain very nearly constant. The intensity of any tone is controlled by a simple resistance network placed between the oscillator and the loud speaker receiver. The current actuating the receiver is

easily calculated from the impedances in the output circuit and from the deflection obtained from a thermocouple in the output of the oscillator. Calibrating measurements have shown that the acoustic energy developed by the receiver is proportional to the electric energy actuating it.[2] Therefore, the rate of emission of acoustic energy from the receiver, or the initial or steady state intensity of a tone sounding in a room, expressed in terms of minimal audibility, can be determined from simple current measurements. Or, if it is desired to produce tones of any standard intensity, for example tones having an intensity equal to one million times the intensity of their minimal audible tones,[3] such tones can be produced by first determining the current through the loud speaker required to produce a barely audible tone and then increasing this current one thousand fold.

Reverberation measurements are made with this apparatus in the following manner: The observer first determines the electric energy required by the loud speaker to produce a barely audible tone of given pitch. Because of the sound interference pattern in the room, it is necessary to make as many as 25 to 50 separate determinations of minimal audibility in all parts of the room and use the average value. The electric energy input to the loud speaker receiver is then increased one million fold, that is, the current through the receiver is increased one thousand fold, and the test tone of standard intensity is thus obtained. The observer then measures the duration of audibility in the room in the usual manner. A double contact key is used, one of which short circuits the loud speaker, thus stopping the source of tone, and the other starts a recording stop clock. The observer holds the key closed until the tone diminishes to the threshold of audibility. When the observer releases the key, the clock is stopped and the tone is started again. This procedure is repeated as many times as the accuracy of the work requires. When precise work is required, 50 separate measurements of the duration of audibility are made for each test tone.

The apparatus has the following advantages: 1. The intensity of the test tone in terms of minimal audibility can be readily determined in any room just prior to the reverberation measurements, by simple adjustments of resistances and the reading of a galvanometer. 2. Since minimal audibility is determined at the time and under the existing

[2] Knudsen, Phys. Review, *21*, p. 89; 1923.

[3] This is the standard intensity of test tones advocated by W. C. Sabine and used by subsequent investigators.

conditions of the test, the measurements of reverberation are less dependent upon residual noises in the room than with the organ pipes, in which latter case the apparatus previously has been calibrated under perfectly quiet conditions. 3. The test tones are practically free from undesirable harmonics. 4. The thermionic tube apparatus is convenient to operate, is portable, and can be set up in thirty minutes.

During the past three years the apparatus has been used successfully to determine the coefficients of sound absorption of many acoustic plasters, tiles, wall boards, and other materials. It also has been serviceable in determining the effectiveness of various acoustic treatments in auditoriums and other architectural enclosures.

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SOUTHERN BRANCH,
LOS ANGELES CALIFORNIA
MARCH 30, 1926.

On the Diffraction of Light Rays by Small Particles. In the ultra-microscope the observation on small particles is made with the axis at right angles to the direction of the illuminating beam. The author discusses his observation that if the particles are viewed in the direction of the incident light, relatively few particles can be observed and these only among the largest. As the angle between the line of view and the incident beam is increased more particles become visible and the intensity of the deflected light becomes greater. At an angle of 150°, about ten times as many particles become visible as at an angle of 90°, which is the usual angle at which the ultra-microscope is used. The author advocates that, in order to observe as many and as small particles as possible with the ultra-microscope, the axis of the latter should be placed at 150 to 160° with the direction of the illuminating beam. The author criticizes the assumption that an illuminated small particle behaves as a minute source of light from which the emitted waves have spherical fronts. He prefers to take the view that the wave fronts are a series of parabolas with the particle at the common focus. [H. Herbst, (Jena) ZS. f. Instrk., *46*, pp. 238–240; 1926.]

PAUL E. KLOPSTEG.

Absorption Spectra of Various Derivatives of Salicylic Acid.—Absorption curves between oscillation frequencies 2000 and 4400 are given for many salicylates and substituted salicylic acids. Change of a radical influences not only the absorption band characteristic of that radical but also the bands due to other parts of the molecule.—[John Edward Purvis; J. Chem. Soc., *129*, pp. 775–8; 1926.]

A.E.R.

A DIRECT CURRENT VIBRATION GALVANOMETER METHOD

By Paul Kirkpatrick

INTRODUCTION

Galvanometer users have, no doubt, often noticed that a small potential admitted to the instrument at properly timed intervals may produce a series of increasing oscillations leading to a maximum deflection many times greater than the steady deflection corresponding to the same applied potential. There exists the possibility that this larger deflection may be of use in the measurement of currents or potentials too small for readable steady deflections. The present paper reports a theoretical and experimental investigation bearing upon the validity of this suggestion and covering the general behavior of d'Arsonval galvanometers when energized in a particular intermittent manner.

THEORY

If a key controlling a galvanometer circuit be periodically opened and closed at intervals of one half the period of the instrument, a suitable potential source being provided, the suspended system will execute a few oscillations of increasing amplitude but will soon attain a maximum which will be maintained for all succeeding oscillations. If a reversing switch be installed in place of the simple key, and the operation be such that a properly directed driving current always flows through the suspended coil, a maximum amplitude larger than in the previous case will be observed. The maximum is found to be proportional to the applied voltage so long as the resistance of the galvanometer circuit is kept constant, and increases with increase of resistance if the current be held at a fixed value. More detailed relations will be developed later.

Fig. 1 is a plot of galvanometer deflections against time, obtained photographically, and shows the motion of the coil both for the ordinary method of use and for the vibration method just described.

After the maximum oscillation has been attained the electrical energy supplied to the galvanometer during any cycle or any half cycle necessarily equals the energy dissipated by the various modes of damping. We proceed to equate them.

The power transformation from electrical to mechanical form within the galvanometer is, as in motors, $P=EI$, where E is the back electromotive force generated in the moving coil and I is the current. Neither E nor I is constant, both being functions of the angular velocity of the coil, ω. In particular $E=k\omega$ and $I=(V-E)/R=V/R-(k/R)\omega$. In these equations V represents the potential difference applied across the galvanometer circuit with its total resistance R, and k is the emf generated in the coil when its angular velocity is unity. The power is then

$$P=\frac{Vk\omega}{R}-\frac{k^2\omega^2}{R} \tag{1}$$

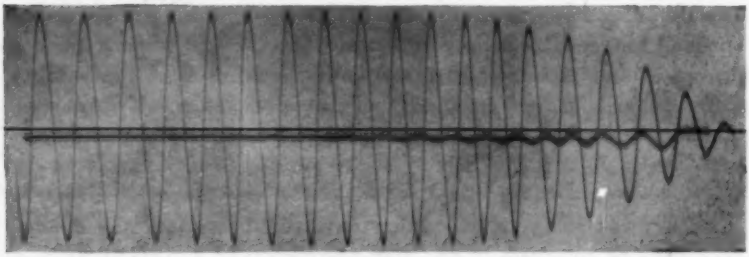

Fig. 1. Photographic record of galvanometer deflections with steady and with periodically reversed applied potential.

Methods of continuous photography showed, in the case of a Leeds and Northrup type R galvanometer, that the coil accurately executed angular harmonic motion, so we may write

$$\omega=\left(\sin\frac{2\pi t}{T}\right)\frac{2\pi\theta'}{T} \tag{2}$$

In this expression θ' is the maximum amplitude and T the period of the galvanometer motion. Inserting (2) into (1) there results

$$P=\frac{2\pi\theta' Vk}{RT}\sin\frac{2\pi t}{T}-\frac{4\pi^2\theta'^2k^2}{RT^2}\sin^2\frac{2\pi t}{T}$$

The total mechanical energy realized in one half cycle or in one swing, as we shall say, is

$$W=\int_0^{T/2}P\,dt=\frac{2kV\theta'}{R}-\frac{\pi^2\theta'^2k^2}{RT} \tag{3}$$

If radians, seconds, and practical electrical units be employed equation (3) expresses W in joules.

The energy of elastic strain in the suspension when the angle of deflection is θ is, by virtue of Hooke's law, $h\theta^2/2$, h being the torque per radian characterizing the suspending wire or strip. Let θ_n and θ_{n+1} be the magnitudes of two successive deflections (opposite in direction) when the coil is oscillating freely with the galvanometer circuit opened. As is well known the ratio, d, of the successive deflections is a constant, independent of the amplitude. The loss of energy in passing from θ_n to θ_{n+1} is

$$\frac{h}{2}[(\theta_{n+1})^2d^2 - (\theta_{n+1})^2].$$

The total displacement during this passage is $\theta_{n+1}(d+1)$. We conclude therefore that in any approximately harmonic swing embracing a displacement $\theta_{n+1}(d+1)$ the above-stated amount of energy will be lost through mechanical damping. If the total displacement during a swing is $2\theta'$ this energy loss may be written

$$2h\frac{d-1}{d+1}\theta'^2,$$

and is expressed in ergs, provided h is expressed in absolute units.

With the galvanometer oscillating at its constant maximum amplitude this loss may be equated to the energy supplied per swing expressed by equation (3), giving, after rearrangement

$$\frac{V}{\theta'} = \frac{h(d-1)}{k(d+1)}R\times 10^{-7} + \frac{\pi^2 k}{2T} \qquad (4)$$

This equation contains no arbitrary or indeterminate constants and is susceptible to experimental test.

EXPERIMENTAL VERIFICATION

These predictions were tested for the case of a Leeds and Northrup type R galvanometer. A variable resistance was connected in series with the galvanometer and the combination supplied with an adjustable reversible potential. Reversals were accomplished by use of a small copper-contact, double-pole, double-throw knife switch, operated periodically by a clock mechanism which in turn was governed by a heavy horizontal pendulum. The pendulum was synchronized with the galvanometer so that their periods agreed to within one tenth of one

per cent. Light from a single lamp filament was reflected from the galvanometer mirror and was brought to focus sometimes on a circular scale and sometimes on the slit of a chronograph arranged for photographic registration. Fig. 1 illustrates the records obtained with the latter conditions. Both curves result from the same potential but with different modes of application, as explained above.

Before comparison can be made between such records as Fig. 1 and the theoretical expectations the galvanometer constants appearing in the equation must be determined. The suspension constant was determined by adding objects of known moment of inertia to the coil and noting the change in period. The constant k was deduced from h and the current sensitivity of the galvanometer, as it is easily shown that the relationship $k=h \times$ (radians per ampere) $\times 10^{-7}$ connects these quantities. The other constants were determined by standard methods and the following findings resulted:

$h =$ .284 dyne cms per radian.
$k = 34.4\times10^{-4}$ volt sec per radian.
$d = 1.124$
$t = 5.40$ sec.

Equation (4) accordingly becomes

$$\frac{V}{\theta'} = 47.9\times10^{-8}R+31.4\times10^{-4} \qquad (5)$$

Fig. 2 presents a series of observed deflections under conditions of varied voltage and resistance. All these observations conform quite closely to an equation

$$\frac{V}{\theta'} = 48.4\times10^{-8}R+34.2\times10^{-4} \qquad (6)$$

In these equations V is expressed in volts, R in ohms, and θ' in radians.

The theoretical discussion is thus well supported. The agreement in the coefficient of R is entirely satisfactory, though the discrepancy between the values of the constant is thought to be larger than the errors of experiment. The larger value found by experiment means that for all resistances and voltages the galvanometer is slightly less sensitive than would be predicted. Investigation showed that this was not a consequence of self-induction, time loss in commutation, or failure of the sinusoidal assumption, but was partially accounted for by non-uniformity of the magnetic field of the galvanometer.

DISCUSSION

Equation (4) reveals at once that high vibration sensitivity is to be obtained by providing a small restoring torque, low damping, low resistance, and a long period. The constant k enters the equation twice, with differing effects. It has a particular optimum value which is a function of the other constants and is given by

$$k_{opt} = \sqrt{\frac{2hRT\,(d-1)\,10^{-7}}{\pi^2(d+1)}}$$

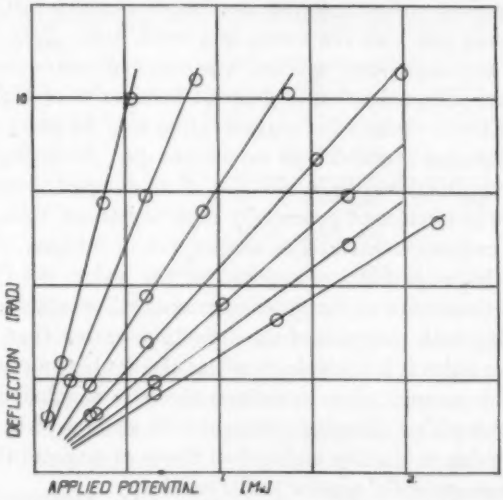

Fig. 2. Variation of vibration deflection with applied potential for seven different values of circuit resistance. The straight lines, taken in clockwise order, correspond to the resistances 4, 10, 20 30, 40, 50, *and* 60 *kilo-ohms, respectively.*

Since the resistance of the galvanometer circuit can not be assumed to remain always at the same figure it is impossible to assign a value for k which shall always be optimum. A value may be stated, however, which is optimum for a probable mean value of the circuit resistance and it happens that k possesses such a value in the case of the galvanometer which was the subject of this study.

The absolute voltage sensitivity attainable by the vibration method is of less interest than the relation between vibration sensitivity and steady current sensitivity. If we denote the steady current deflection by θ'' we have for the steady current sensitivity

$$\theta''/V = k \cdot 10^7/Rh.$$

The ratio of vibration sensitivity to the ordinary sensitivity may be called the magnification, and denoted by the letter M. The possible utility of the vibration method depends upon the attainable values of M.

$$M = \frac{\theta'/V}{\theta''/V} = \frac{1}{\frac{d-1}{d+1} + \frac{\pi^2 k^2 10^7}{2RTh}}$$

High magnification is to be found with galvanometers of low damping, long period, stiff suspension, and small k, which last requirement means a small coil with few turns, in a weak field. Only the first two conditions may be imposed without impairment of one or the other of the sensitivities. Magnification is greatest in circuits of high resistance. In low resistance circuits the magnification may be unity or even less, and the vibration method is of no advantage. Accordingly the most favorable applications lie in the use of undamped, long-period galvanometers in circuits of necessarily high resistance, such as occur in insulation measurements and in some types of bridges. As resistance increases, the magnification approaches the value $(d+1)/(d-1)$, regardless of the values of the other instrumental constants.

By reading both extremes of the deflections rather than one extreme and the zero point it is possible to utilize the double amplitude, and so, in effect, the magnification as defined above is multiplied by two. A galvanometer with a damping ration of 1.05 when used in this manner yields a maximum reading eighty-two times as large as that obtained from the same potential applied in the usual way.

A question of importance is that which concerns the time occupied by the growth of the deflections to their maximum. This matter may be investigated by considering the properties of the envelope of the large-range oscillatory curve of Fig. 1. Any maximum of the oscillatory record represents an angular deflection of the galvanometer system of $y/2D$, y being the ordinate of the enveloping curve and D the distance from the galvanometer to the recording paper. The potential energy of elastic strain in this condition is $W = hy^2/8D^2$. The net mechanical energy acquired by the galvanometer system in one swing from right to left or vice versa is the total energy derived from the electrical circuit diminished by the amount of energy dissipated through damping.

$$\Delta W = \left(\frac{kV10^7}{R}\right) A - \left(\frac{\pi^2 k^2 10^7}{4RT} + \frac{h(d-1)}{2(d+1)}\right) A^2.$$

or, more concisely,

$$\Delta W = PA - QA^2,$$

where by A we represent the full angular swing of the coil.

Since the duration of the swing is $T/2$, and since $A = y/D$, we have approximately

$$\frac{dW}{dt} = \frac{Py/D - Qy^2/D^2}{T/2}$$

Substituting the value of W given above and integrating there results the equation of a smooth curve enveloping the upper branch of the oscillatory record:

$$y = \frac{PD}{Q}\left(1 - e^{\frac{-8Qt}{kT}}\right)$$

or

$$y = \frac{1 - \exp.\ \frac{4t}{T}\left(\frac{10^7\pi^2k^2}{2RTh} + \frac{(d-1)}{(d+1)}\right)}{\frac{1}{kVD}\left(\frac{\pi^2 k^2}{4T} + \frac{hR(d-1)10^{-7}}{2(d+1)}\right)}$$

This equation fits the extremes of Fig. 1 and similar records sufficiently well, and may be applied to show that the present galvanometer in a 20,000 ohm circuit, attains 99 per cent of its maximum in fifteen complete oscillations. The same galvanometer, connected in the usual way, after fifteen oscillations would still have an amplitude of three per cent of its ultimate deflection.

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A Principle of Correspondence.—Using electrons of homogeneous velocities, critical potentials of Hg vapor are found at 10.4, 10.6, 11.2, 11.6 and 11.9 volts. The probability of ionization is finite at 10.4 volts, decreases to 10.6, rises suddenly at 10.6, and repeats this behavior at the other potentials. The probability, as a function of the energy of the impacting electrons, resembles the probability of ionization by light or by x-rays as a function of the frequency of the incident quantum. This suggests that electrons and quanta obey the same general laws in ionization processes. Thus Bohr's theory of ionization by beta rays may be applied with little change to predict the probability of scattering of x-ray quanta (Compton effect) as a function of frequency.—[Ernest O. Lawrence; Science, *64*, p. 142; 1926.]

A.E.R.

The Glow Light Oscillograph. The article gives a summary of investigations which have been made in the application of the glow discharge as means for investigating the wave form of an alternating current. The original glow discharge tube of Gehrke consists of coaxial electrodes of thin nickel rod, separated by an interval of several millimeters and sealed into a glass tube having a suitable kind of residual gas to produce the desired color in the glow. H. A. Wilson discovered that the extent of the glow on the electrodes is proportional to the intensity and direction of current. When the discharge is observed with a rotating mirror the wave form of the alternating current appears, but the halves of the wave are separated by the width of the gap between the electrodes. In a modified form of the tube this difficulty is overcome by placing the electrodes on parallel axes separated by a small interval and having their free ends terminating in the same plane at right angles to the axes. Other developments are touched upon and the final design due to Gehrke and the author is described. In this design the discharge is made to occur between two metal strips separated by a small interval and attached to the same electrode. To obtain the full wave of the current being investigated the latter is superposed on a direct voltage. The magnitude of this voltage determines the zero line of the wave. A discussion follows of the application of the glow light oscillograph. It is pointed out that it is possible to follow a frequency of 11,000 per second with ease, and that with a frequency of 680,000 per second it was possible to observe individual cycles but not wave form. For many investigations the glow light oscillograph appears to be as suitable as the cathode ray oscillograph. One disadvantage is that it requires 30 to 40 m.a. for operation. To overcome this disadvantage the author describes an amplifying circuit. A bibliography of twenty-three references is appended. [Viktor Engelhardt (Berlin) ZS. f. Instrk., *46*, pp. 240–248; 1926.]

PAUL E. KLOPSTEG.

On Some Laws of Optical Spectra.—In order to apply the displacement law of Kossel and Sommerfeld in complicated spectra, the authors modify it as follows: In comparing an arc spectrum with the first spark spectrum of the succeeding element, we may expect similar properties for those groups of terms which have the same multiplicity and which arise from the same set of k-values. Thus, separations of multiplet terms which fulfill this criterion are frequently in a constant ratio for a pair of adjoining elements, though the ratio alters if we pass to other pairs. Deviations from Lande's interval rule are in the same sense for corresponding multiplet terms. Heisenberg has shown that in 2-electron systems the multiplet separations of terms having the same l's and k's should be inversely proportional to their multiplicities. The authors extend this result to systems having any number of electrons.—[Karl Bechert and M. A. Catalán; Anales soc. españ. fis. quim., *24*, pp. 222–33; 1926.]

A.E.R.

THE DISTRIBUTION OF LIGHT INTENSITY IN ASTIGMATIC IMAGES, WITH SPECIAL REFERENCE TO THE SENSITIVITY OF TEST CHARTS USED IN REFRACTION

By Jonas S. Friedenwald

Modern studies in geometrical optics are concerned chiefly with complicated problems in the construction of optical instruments, involving the analysis and methods of correction of the aberrations of the higher orders. Problems relating to such elementary matters as axial astigmatism seem to have been largely neglected, probably because axial astigmatism rarely needs to be considered in the design of optical instruments. However, the widespread occurrence of astigmatic errors in the eyes of human beings compels us to investigate this subject. It may be worth while, therefore, to attempt an analysis of the exact nature of images that cannot be sharply focused in consequence of a direct axial astigmatism. In doing this it seems justifiable to neglect the effects of diffraction and aberrations of the higher orders and to limit the discussion to first order optical theory, though in no actual physical case will the image be exactly as we shall describe it.

Among the questions that present themselves are the following: In what manner is the light arising from a point in the object distributed in the image plane? In what way are object forms of various sizes and shapes represented in the image? What shape and character of an object will be most poorly represented in the image? This last question has a practical significance in that its answer may aid us in determining the best form of a test-object for the subjective determination of the astigmatism of the human eye.

The method to be pursued in this paper is a simple extension of that developed by Helmholtz[1] in determining the nature of images which are blurred by imperfect accommodation or by the chromatic aberration of the eye.

In an astigmatic optical system such as we are considering there are two image planes corresponding to any object plane. For each of these image planes we can pick out a set of lines in the object plane which will be well depicted in the image; and every point in the object plane

[1] Helmholtz, Physiological Optics, 3rd Edition, *I.* p. 156 (German Edition), p. 183 (English Edition).

will be represented by a short segment of that line in the image plane which is the image of the line in the object plane in which the point lies. What we have to consider is in what way the light arising from a point in the object plane is distributed along this short segment of the image line, and from the solution of this problem we shall attempt to reconstruct the images of object forms of various sizes and shapes. Since the object plane may be considered to be made up of a system of parallel imageable lines, for our purposes any object will be adequately represented by the system of intercepts which the object makes on these imageable lines. Such intercepts may then be further divided into segments over which the light intensity from the point of view of the image-forming system is sensibly uniform. Such segments may be small or large when considered in relation to the resolving power of the astigmatic system and it is with the representation of such linear segments in the astigmatic image that we shall be concerned. The analysis of the more general case in which the light intensity is continuously variable along an intercept of one of the imageable lines in the object plane introduces mathematical problems which are, in general, not directly soluble and which can only be attacked by graphical methods of integration. The problem which we are attempting to solve may, therefore, be re-defined as follows:

To find the distribution of light intensity along an image-line in an astigmatic system corresponding to a long or short, or infinitely short, uniformly luminous segment of an imageable line in the object plane. The simplest object forms which will be represented, respectively, by long or short or infinitely short segments of imageable lines are an area of appreciable size, a narrow line, and a point, and we shall speak of the astigmatic images of these three object forms in order to avoid the more cumbersome expression of "uniformly luminous segments of imageable lines in the object plane," though no further limitation of the problem is to be implied by this circumlocution.

CASE 1.—THE ASTIGMATIC IMAGE OF A POINT

In an astigmatic optical system the image of a point consists of two short linear segments one in each image-plane, and each one of these image-lines is parallel to one or the other of the axes of astigmatism of the optical system. (Fig. 1)

Let CD be one of these image lines of length $2r$;

AB the exit pupil of the optical system of diameter $2R$;

f_1 the distance from the first image-line to the plane of the exit pupil;

and f_2 the distance from the second image-line to the plane of the exit pupil.

The first image-line is not shown in the figure since it must be perpendicular to the plane of the paper. It crosses the plane of the paper at the intersection of the lines AD and BC.

Light reaches the point x not only from y but also from all points along a perpendicular to AB at y in the pupillary zone, as represented by PQ in Fig. 2. That this is true can readily be seen by imagining the projection of the first image line upon the plane of the exit-pupil with x as origin of the projection. If the illumination throughout the pupillary zone is constant the intensity at x will be proportional to the length of PQ.

FIG. 1

FIG. 2

Assuming the pupil to be round and measuring x and y from the axis of the optical system, we may write:

$$I = H(PQ)$$
$$= 2H\sqrt{R^2 - y^2} \qquad (1)$$

where I is the intensity of illumination at x and H denotes a constant depending on the intensity of illumination of the pupillary zone..

Since R, r x, y are related in the following manner:

$$\frac{y}{x} = \frac{R}{r} = \frac{f_1}{f_2 - f_1},$$

we may write the above equation in the form:

$$I = 2H\sqrt{\frac{r^2 f_1^2}{(f_2 - f_1)^2} - \frac{x^2 f_1^2}{(f_2 - f_1)^2}} \qquad (2)$$

For the first image-line the equation would be

$$I = 2H\frac{f_2}{f_2 - f_1}\sqrt{r^2 - x^2} \qquad (2a)$$

Since f_1 and f_2 are to be considered as constants we may write:

$$I = 2aH\sqrt{r^2 - x^2} \qquad (3)$$

The graphical representation of this equation is shown in Fig. 5.

CASE 2.—THE BLURRED EDGE OF THE ASTIGMATIC IMAGE OF A UNIFORM AREA

Let us suppose that there exists in the object-plane a uniformly luminous area of sufficient size that the width of its image in the absence of astigmatism would be greater than r. The central part of this image will then, be of a uniform luminosity, but the edges of the image, unless they are parallel to one of the axes of astigmatism (which axis will depend upon which image-plane is chosen for investigation) will be blurred. In what way does the intensity of illumination vary across this blurred fringe?

Let AB (Fig. 3) be the edge of the image in the absence of astigmatism, and CD one of the image-lines of this image-plane.

In the absence of astigmatism a point in the image such as P receives all the rays of light entering the optical system which emanate from a corresponding point in the object. When astigmatism is present, these

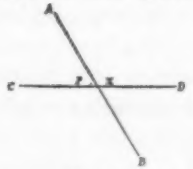


FIG. 3

FIG. 4

rays of light are distributed along a short segment of the line CD, whose mid-point is P and whose length is $2r$. Thus, on the assumption of the existence of astigmatism, a point x will receive light from any point in the object along a line in the object of which CD is the image provided that the distance from x to the mid-point P of the image-line, corresponding to this object point, is not greater than r. The intensity at x due to any single object-point such as that corresponding to P will be given by equation (3). To get the effect of all possible object-points, this equation must be integrated between the values x and r, for if we take the intersection of AB and CD as the origin the distance between x and P cannot be less than x nor greater than r.

$$J = \int_x^r I\,dx = \int_x^r 2aH\sqrt{r^2-x^2}\,dx$$

$$= 2aH\int_x^r \sqrt{r^2-x^2}\,dx$$

$$= 2aH\left[\frac{r^2}{4} - \frac{x}{2}\sqrt{r^2-x^2} - \frac{r^2}{2}\sin^{-1}\frac{x}{r}\right] \qquad (4)$$

This formula is represented graphically in Fig. 6.

CASE 3.—THE ASTIGMATIC IMAGE OF A SMALL AREA OR NARROW LINE

Let us suppose that there exists in the object plane a uniformly luminous area so small that the width of its image as measured along one of the image-lines would, in the absence of astigmatism be less than r.

Let $2d$ (Fig. 4) be the width of the image in the absence of astigmatism measured along one of the image-lines ($2d < r$).

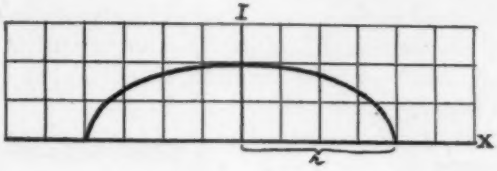


FIG. 5

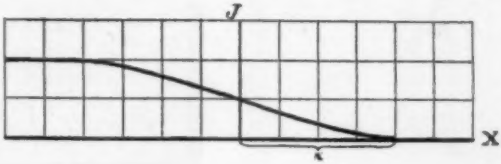


FIG. 6

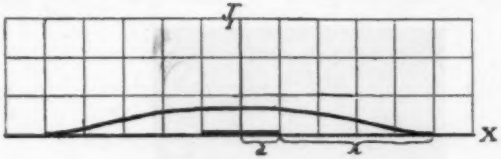


FIG. 7

Let x be any point on this image-line. For values of x greater than $(r-d)$ or less than $(d-r)$ the intensity can be obtained from equation (4) by means of obvious transformations. Between $\pm(r-d)$ the following integration will be seen to hold

$$J_1 = \int_{x-d}^{x+d} I\,dx = \int_{x-d}^{x+d} 2aH\sqrt{x^2-r^2}\,dx$$

$$= 2aH\left[\frac{x+d}{2}r^2 - (x+d)^2 + \frac{r^2}{2}\sin^{-1}\frac{x+d}{r}\right. \tag{5}$$

$$\left. - \frac{x-d}{2}r^2 - (x-d)^2 - \frac{r^2}{2}\sin^{-1}\frac{x-d}{r}\right]$$

When d becomes very small this formula reduces to

$$J_2 = 2a\,d\,H\,\sqrt{r^2 - x^2}\,.$$

A graphic representation of the case $d = r/4$ is shown in Fig. 7.

These formulae are sufficient to enable us to reconstruct graphically the images of some objects as they appear to persons whose eyes have an astigmatic defect. In particular it seemed worth while to apply this method to the study of the images of the different types of astigmatic dials which are used in the subjective measurement of astigmatism. The charts which we have compared are those of Lancaster, Wallace, and the writer.

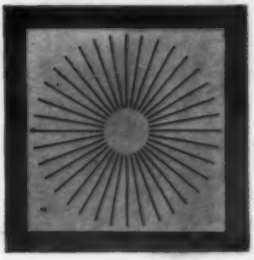


FIG. 8. *The Lancaster chart.*

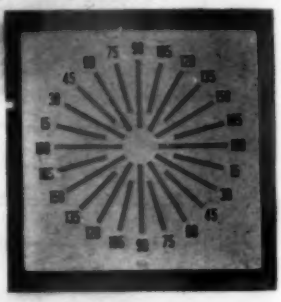


FIG. 9. *The Wallace Chart.*

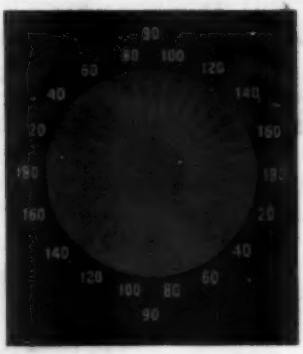


FIG. 10. *The author's chart.*

It must be clearly stated at the outset that no mathematical analysis, however complicated, can determine the practical usefulness of a clinical method. The ultimate test of practical usefulness is experience;

but it is possible, we believe, to render a little more clear just what is actually happening when these clinical methods are applied. Especially in the subjective method of refraction, it seems important that we

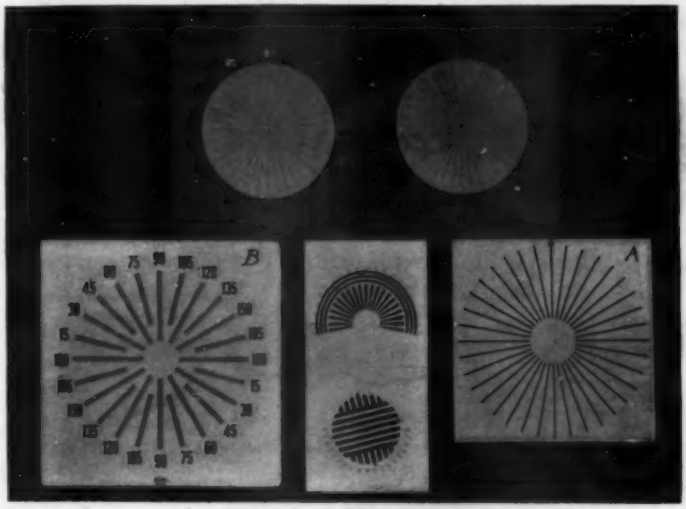
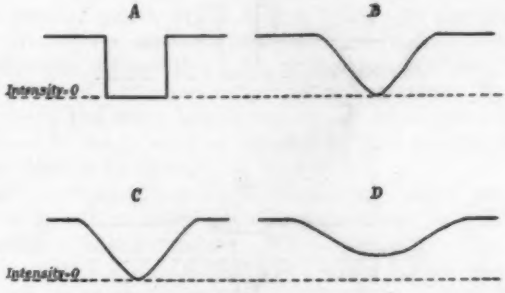


FIG. 11. *Set of charts photographed with an astigmatic camera equivalent to a myopic astigmation of 0.25D for the human eye. A; Lancaster chart. B; Wallace chart. C; authors' chart.*

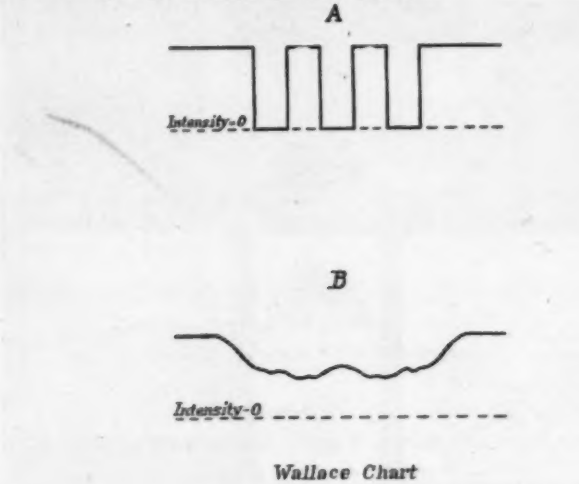


Lancaster Chart

FIG. 12

A. Distribution of light intensity in the image of the chart when clearly seen.
B. Same when seen with 0.25D astigmation.
C. Same when seen with 0.25D fog.
D. Same with both fog and astigmation.

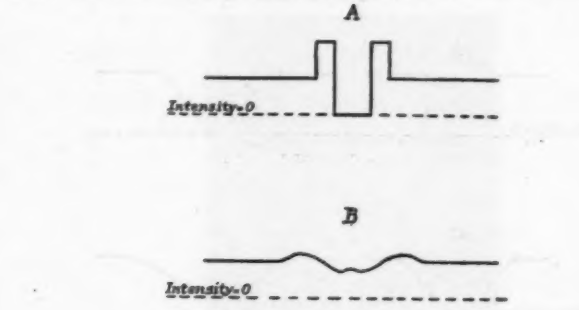
should have as clear a knowledge as possible of just what images we are asking the patient to discriminate, so that we may better realize how much we tax his judgment, and so that we may have little more understanding of his difficulties and errors.



Wallace Chart

FIG. 13

A. Distribution of light intensity in the image of the chart when clearly seen.
B. Same when seen with 0.25D astigmatiion.



Friedenwald Chart

FIG. 14

A. Distribution of light intensity in the image of the chart when clearly seen.
B. Same when seen with 0.25D astigmation.

Figs. 8, 9, and 10 show the three charts which are to be compared. The remaining graphs show the distribution of light intensity across an

image of the lines of these charts when they are seen by an eye with a myopia astigmatism of one-quarter diopter and a pupil of four mm. diameter, the edge of the lines being perpendicular to the direction of the astigmatic blur. For the Lancaster chart the condition is portrayed as it would appear with and without a fog of one-quarter diopter, since Dr. Lancaster has designed his chart particularly to be used with such a fog.

In comparing the graphs Figs. 11 to14, the reader must remember that what the patient is asked to judge is not the difference between the blurred images of the various charts but the difference between the clear and the blurred lines on a particular chart, and that the best chart is that in which the difference between the clear and blurred lines will be most apparent to a patient of average intelligence.

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JOHNS HOPKINS MEDICAL SCHOOL,
BALTIMORE, MARYLAND.

Table of the chief Lines of the line Spectra of all Elements arranged according to Wave-lengths.—By H. Kayser; VII+198 pages; Julius Springer; bound 24 Reichmark ($6.00).

The sixth volume of Kayser's handbook, published in 1913, contained a table of the stronger spectral lines arranged according to wave length. The present book represents a complete revision and extension of this early work, in which are incorporated a summary and critical review of the astonishingly vast amount of new material published during the past 13 years, up to March 1926. On the whole, Kayser states, the precision of measurement has been pushed to one more significant figure and for possibly 10% of the stronger lines, to two more significant figures i.e., to the thousandth Angstrom. In addition to higher precision the wave length region has been extended by many measurements in the infared to 1μ, and by four octaves in the ultraviolet, from 2000 A to 125A.

The table contains about 19000 lines, each, when possible, representing a weighted mean of all determinations, in general good to ± 3 in the last significant figure. The largest wave length tabulated is 9.0850μ for NaI and the shortest $124.0A$ due to Ga. The intensity of each line and its special characteristics such as diffuse, very diffuse, asymmetrically broadened, self-reversed, and structure, when too fine to be separately resolved, are included. The uncertainty of the intensity estimates is clearly explained in the preface; at present the numbers can serve only as a guide for the spectroscopist and cannot be taken very seriously. All wave lengths are expressed in International

Angstroms and a table for converting from the Rowland to the I. A. scale is presented for those who must refer to the earlier publications.

The author points out that in a field which is developing at rate characteristic of spectroscopic research, any table which is satisfactory today, within six months will require considerable revision. For this reason the book has been printed on a very fine grade of writing paper with blank margins of from 1 1/2 to 2 inches surounding each page of data upon which the reader has plenty of room for inscribing the necessary corrections.

This is the second monumental work within a year, coming from Prof. Kayser, the first, as he has expressed it, representing the spare time of his increasing age. Every spectroscopist must have both books upon his desk for constant reference; and incidentally as an object lesson, pointing out what can be accomplished by a single individual when he indefatigably applies himself to the task at hand.

P.D.F.

Chemical Reactions and Volumetric Titrations in Ultraviolet Light from a Wood Lamp.—Many reactions occurring in solution are accompanied by changes in the intensity or color of fluorescence emitted by the reacting substances or by fluorescent materials added as indicators. The authors study hydrogen ion concentration by the aid of quinine, which is fluorescent in acids but gives no light in basic solutions. In most of the cases studied, the accuracy compares favorably with that obtained by the usual indicators in white light. The fluorescence excited by ultraviolet light is a surface phenomenon in many liquids, so that the method is applicable to turbid or to strongly colored liquids.—[R. Mellet and A. Bischoff; Compt. Rend., *182*, pp. 1616–9; 1926.]

A.E.R.

Absorption of Beta Rays by Matter.—Measurements of mass absorption coefficients μ/ρ of liquid and solid compounds and compounds dissolved in water. For the following, μ/ρ is compounded additively out of the μ/ρ values for the atoms of the compounds: SiO_2, MnO_2, MgO, Al_2O_3, HBO_3, KBr, $NaNO_3$, KNO_3, $HKSO_4$, naphthalene, picric acid, CCl_4, CS_2. The following have μ/ρ values 8 to 28% greater than those calculated from the additivity relation:

H_2O, ZnS, CaF_2, $Fe(CN)_6K_3$, urea, camphor, $CaSO_4 \cdot 2H_2O$, oxalic acid $(2H_2O)$, $As_2S_3 \cdot 6H_2O$, potassium alum $\cdot$ 24 H_2O, and aq. solutions of KBr, KI, NaBr, NaI.—[Georges Fournier, Compt. Rend., *183*, pp. 200–3; 1926.]

A.E.R.

ASPHERIC ENLARGING CONDENSER

By L. V. Foster

Fig. 1 shows diagramatically the arrangement of light source, condenser, negative, lens and easel used in the enlarging of photographic negatives. One of the problems of enlarging consists in illuminating the negative in such a way that the enlarged photograph will have the same grades of density as exist in a contact print from the original. This requires not only that the negative be evenly illuminated but

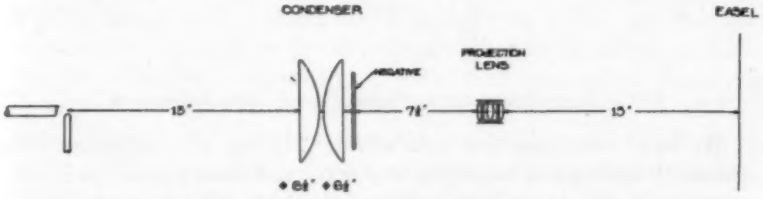

Fig. 1. *General optical arrangement for making enlargements.*

also that all the light which passes through a given point of the negative be transmitted by the enlarging lens. To the first order of approximation this is accomplished when the condenser images the light source in the diaphragm plane of the enlarging lens and the negative is placed close to the condenser.

As can be seen from Fig. 1, the condenser is somewhat larger than the negative. This is because of two reasons. In the first place, it is impractical to put the negative too close to the condenser, for dust and dirt on the condenser surface would appear in the enlargement. Second, the cone of light emerging from the condenser is not of uniform density in cross section but is reasonably so if the outer part of the cone be eliminated. This can only be done by making the condenser so large that the negative occupies only the reasonably homogeneous portion of the cone of light. The lack of uniformity of illumination in the cross section of the beam of light is due to spherical and chromatic aberration in the condenser. Of these the spherical is the chief offender. In addition to the inequality of illumination it also results in more or less of the light which passes through the marginal parts of the negative failing to pass through the enlarging lens as shown in Fig. 2.

When enlargements are attempted of larger size negatives, e.g. 8×10, the diagonal of which is $12\frac{3}{4}''$, the minimum size condenser is 14″ in diameter. An 8×10 negative, however, placed sufficiently remote from the condenser to eliminate any possibility of reproducing dust particles or physical defects on the condenser surface, is not illuminated evenly. With the best adjustment of illumination, the corners of the negative are not covered in the enlargement.

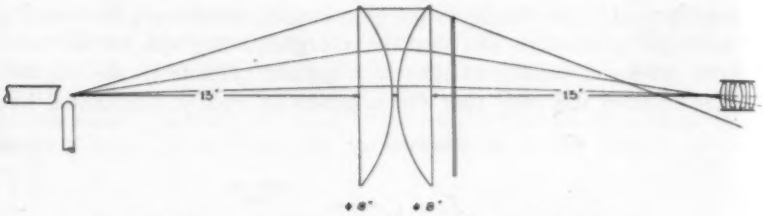

FIG. 2. *Spherical condenser showing the effects of spherical aberration.*

We have overcome this difficulty by the use of condensers with parabolic surfaces by means of which the spherical aberration is corrected. The usual type of condenser consisting of two plano convex lenses facing each other was replaced by two lenses as shown in Fig. 3.

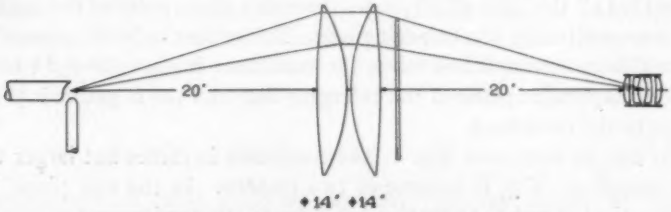

FIG. 3. *Aspheric condenser corrected for spherical aberration.*

Both elements of the condenser are alike and have one parabolic surface, which combined with a spherical surface, corrects the spherical aberration to such an extent that a 14″ condenser illuminates perfectly the entire surface of an 8×10 negative. This is a further application of the principle of combining spherical and parabolic surfaces described by Mr. W. B. Rayton at the Washington meeting of this society in 1922 in a paper on "The Practical Application of Parabolic Surfaces in Lens Construction." We have supplied such condensers for enlarging negatives taken in aerial surveying where they have given complete satisfaction.

It can be readily appreciated that the difficulty in making enlargements is accelerated directly with an increase in the size of negative. By this, I mean that there is a limit to the size of uncorrected condenser which can be used for enlarging negatives. This limit is practically 14″. The 14″ uncorrected spherical condenser will not furnish illumination sufficiently even for enlarging an 8×10 negative. Thus, the 14″ aspheric consdenser described above was designed for this purpose. There is, however, a limited field for making enlargements from negatives still larger than 8 × 10. Not long ago we were asked by a certain lithographing concern to construct condensers which would permit the enlarging of half tone screen negatives 18″ square. They wanted to make enlargements 6 feet high for special bill-board posters and backgrounds for window displays. If a single direct enlargement 6 feet high is made from an 8 × 10 negative the silver grain becomes magnified so much that the picture is worthless. They found that this could be overcome by an intermediate enlargement through a half tone screen to an 18″ size. This half tone negative, when enlarged to the required size, gave entire satisfaction. For this work we made a condenser system composed of two hollow condensers each consisting of two glass shells, filled with water, the inner shells being parabolic in form (Fig.4). This

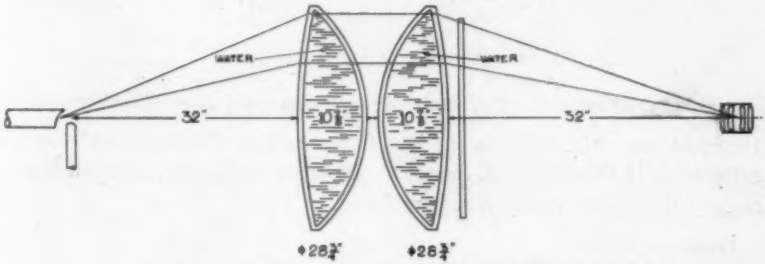

Fig. 4. *Water cell condenser corrected for spherical aberration.*

condenser was made 28¾″ diameter and because of its correction of spherical aberration covers completely the 18″ square plate the diagonal of which is 25½″. An enlargement 6 feet square made with one of these condensers was exhibited at the time when this paper was presented before the society.

Fig. 5 gives a table showing the distances between condenser, enlarging lens, and screen for various magnifications. It can be seen that work of this nature requires special equipment not only in handling the paper for development but a specially large dark room for making

MAGNIFICATION	ARC TO CONDENSER	CONDENSER TO ENLARGING LENS	ENLARGING LENS TO EASEL	ARC TO EASEL
1	23.5 INCHES	50.0 INCHES	50.0 INCHES	12 FEET
4	32.0 "	32.0 "	130.0 "	18 "
6	35.5 "	29.0 "	182.0 "	22.5 "

Fig. 5. *Dimensions for enlarging with water cell condenser.*

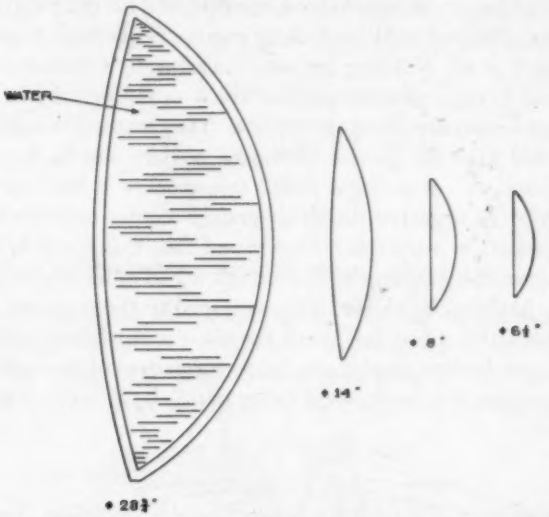

Fig. 6. *Comparative sizes of standard enlarging condensers and water cell condenser.*

the exposure. Fig. 6 shows one lens from the four different condensers referred to in this paper all drawn to the same scale. The comparative sizes of them can readily be appreciated.

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Fluorescence of Bromine Vapor.—At pressures inferior to 1 mm of Hg sunlight excites a green fluorescence in Br_2 vapor. By the use of filters it was shown that the most effective wave lengths lie between 5000 and 5600 A, in the molecular absorption spectrum which extends from about 5000 A to the infrared. Wave lengths in the continuous band below 5000 A excite no fluorescence. A spectrogram of the fluorescent light obtained with very small dispersion showed the presence of radiations between 5000 A and the red, with center of gravity shifted toward the red as compared with the incident light.—[P. Daure; Compt. Rend., *183*, pp. 31-3; 1926.]

A. E. R.